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CONTENTS

Vermilye Medal Presentation 1
Management and the Free Market ..... J. HOWARD PEW 5
Specific-Surface Determinations of Uranium Dusts by Low-Temperature Adsorption of Ethane ..... KENNETH E. LAUTERBACH, SIDNEY LASKIN AND LEONARD LEACH 13
An Extension of a Theory of Magnetic Amplifiers ..... ROBERT T. BEYER AND MING-YI WEI 25
Zirconium as a Material for Fractional Weights ..... WILLIAM M. THORNTON, JR. AND EDWARD S. HAUBER 39
An Electronic Integral-Transform Computer and the Practical Solution of Integral Equations ..... HENRY WALLMAN 45
Notes from the National Bureau of Standards 63
The Franklin Institute
- Annual Report of the Board of Managers for the Year 1949 67
- Museum 76
- Committee on Science and the Arts 77
- Membership 77
- The Franklin Institute Laboratories for Research and Development 78
- Journal of The Franklin Institute 79
- Library 79
- Bartol Research Foundation 82

Book Reviews 84
Book Notes 87
Notes from the Biochemical Research Foundation 89
Current Topics 38, 62, 88, 91

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Journal
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Devoted to Science and the Mechanic Arts

Vol. 250 JULY, 1950 No. 1

VERMILYE MEDAL PRESENTATION

The Franklin Institute, Wednesday, May 17, 1950

At the presentation of the Vermilye Medal, made at the Stated Meeting of The Franklin Institute, held May 17, 1950, in the Lecture Hall, Mr. Richard T. Nalle, President of the Institute, recognized Mr. Ralph Kelly, member of the Board of Managers and Chairman of the Vermilye Medal Committee. Mr. Kelly, in presenting Mr. J. Howard Pew for the award, spoke as follows:

"Mr. President:—It is my privilege as Chairman of the Vermilye Medal Committee of The Franklin Institute to tell you briefly about the medal and to present the recipient to you.

"William M. Vermilye was an eminent industrialist and banker and a great friend of the The Franklin Institute. He was a descendant of Johannes Vermilye, an early Manhattan settler. It was his idea to endow a medal for industry to be awarded no more frequently than biennually by The Franklin Institute, this medal to require the same high qualifications for industry as those for the Franklin Medal in the scientific field. It is stated that the recipient should have the homely wisdom of Poor Richard and should share in his desire to serve humanity.

"The Vermilye Medal was last awarded in 1946. It will be interesting to you to read the names of the distinguished men who have received this medal since it was instituted:

Lewis H. Brown	1939
Lt. Gen. Wm. S. Knudson	1941
Walter S. Gifford	1943
Martin W. Clement	1946

To this illustrious group is added the recipient who is with us tonight.

* Proceedings of the Vermilye Medal Award, presented to Mr. J. Howard Pew, May 17, 1950, at The Franklin Institute.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

"Although he is well-known to all of you—to all of Philadelphia and to the Nation—it is not amiss to cover a few high spots in his career. He was born at Bradford, Pa., and early in life had the great wisdom and good fortune to marry Miss Helen Jennings Thompson of Pittsburgh. You will note that I gave that the place of first importance. Some time before entering the state of matrimony—at the age of 19 to be exact—

Photo courtesy of Sun Oil Company

Mr. J. Howard Pew (left) receives the Vermilye Medal from Mr. Richard T. Nalle.

he entered the employ of the Sun Company at Marcus Hook as an engineer at the refinery. At a remarkably early age, in 1912, he became President of that company—a post he held for thirty-five years. During his presidency the company grew from an annual volume of 5 million dollars to over 356 million dollars. His influence on the petroleum industry was just as great. In approximately the same period, petroleum products as a source of national energy increased from 15 to 51 per cent,

while in the same period coal as a source of national energy declined from 82 to 46 per cent.

"His great contribution to the petroleum industry was recognized last year by the award of the Gold Medal for Distinguished Achievement of the American Petroleum Institute.

"He has found time in his busy life to forward the interests of the Boy Scouts and in this activity, as in all others, he never does anything half way. He is just as interested and as much of a factor in a small troop as he is in the national council.

"He is President of the Board of Trustees of Grove City College and also President of the Trustees of the General Assembly of the Presbyterian Church in the United States. To one who does not know his versatility or his power of concentration, it is difficult to understand how he can devote a large amount of time to church activities and still so consistently play such a good game of week-end golf. But he does and when you play against him, prepare to play better than you know how or you will take a sound beating.

"I could sum up his character and achievements by saying that he is of the same sturdy clear thinking type as Herbert Hoover. This type of thinking is needed so badly in this post-war period to re-establish true values in the minds of the people and guide our actions by a greater degree of spiritual idealism.

"Some of us were present at the dinner inaugurating the Savings Bond Drive. One of the most impressive sidelights of that evening was the statement made by an actor—a song and dance man—who said that he was raised in poverty on the west side of New York, but rose to a top place in his profession without hurting anyone. That is the kind of an American Howard Pew stands for.

"I know you will be interested in the reading of the citation:

> In recognition of his outstanding contribution to the science of industrial management through his leadership in an industry that is essential to our life and which now supplies more than fifty per cent of the energy used by the American People; through his encouragement and support of scientific research which has resulted in great advances in the fields of technology, health, and management, thus helping to preserve the American way of life; and through his steadfast adherence to the principles on which our Country was founded and which have made it great.

"Mr. President, it is my great privilege to present to you the recipient of the Vermilye Medal for 1950—Mr. J. Howard Pew."

Mr. Nalle, by virtue of the power vested in him as President of The Franklin Institute, handed the medal and accompanying certificate to Mr. Pew with his congratulations.

After thanking Mr. Nalle, Mr. Pew delivered his address, "Management and the Free Market." This talk (see this JOURNAL, p. 5), served the dual purpose of acknowledging the receipt of the Vermilye Medal and of honoring the memory of Charles Day.

MANAGEMENT AND THE FREE MARKET*

BY

J. HOWARD PEW[1]

It is quite impossible for me to express in adequate language my appreciation of this high honor.

We have it from the Scriptures that a prophet is without honor at home. And so it becomes a source of very real satisfaction to me when I realize that the members of the Board of this great institution are among those with whom I have been associated down through the years.

As I stand here tonight amidst this assemblage of scientists, within the walls of one of the most famous of all scientific institutions, I am reminded of an old hymn of our childhood days which I am sure many of you recall. It ran something like this:

"I want to be an angel and with the angels stand." Well, that was the way I felt about science as a young man. My greatest ambition was to be a scientist and "with the scientists stand." But the relentless hand of fate decreed that I should spend most of my active life in the field of management. Now, after almost a half century of active managerial work, I find myself "standing with the scientists" here tonight and receiving at their hands one of their most coveted awards. While I accept this award with the utmost humility, I cannot help but wonder objectively how such a thing could happen. Has management come so far along the road of progress and enlightenment that it has earned a place in its own right in the field of science, or has science become so advanced and liberal in its thinking that it recognizes management as a phase of science? I would like to believe that both processes have brought this about. At any rate, I am very happy to be "standing with the scientists" tonight.

When Dr. Allen advised me of this award, he told me that I was expected to respond and assigned to me "Management" as my subject. At the time I was far from sure that I had anything to say which would be both appropriate and interesting. Then it occurred to me that if I accepted the assignment literally and discussed management—its historical development, its present status and its future prospects—I might bring out some interesting facts.

The struggle for Anglo-American liberty had its beginnings back in 1215 A.D., when the barons marched their armies down to the banks of

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* Presented under the Charles Day Lecture Foundation of The Franklin Institute of the State of Pennsylvania, given in the Lecture Hall of The Franklin Institute at the Stated Meeting held May 17, 1950. The Franklin Institute's Vermilye Medal for leadership in the field of industrial management was awarded to Mr. Pew on this occasion.

[1] Director and former President, Sun Oil Company, Philadelphia, Pa.

the Thames at Runnymede and compelled King John to sign the Magna Charta. Up until that time the peoples of the world had lived under various schemes of government control and economic planning. Their very lives and activities had been subjected to the whims and foibles of their bureaucratic overlords. There were a few brief periods—such as those which obtained in the early development of Greece and of Rome—in which there was established a measure of liberty.

But with the exception of these brief periods, industry had been conducted on a very elementary scale, largely dominated by the government. The comparatively few people engaged in industry were mostly serfs of the state or industrial slaves. But most of the people not directly employed by the government, were engaged in agriculture, where they struggled with a few hand tools and produced barely sufficient food to eke out an existence for themselves and their families. It can readily be seen, therefore, that under such conditions there existed no field in which management could operate.

For the next 500 years there was a perpetual quarrel between the kings and the Parliament. During certain periods Parliament would succeed in exercising a measure of control over the king; and during other periods the king would make Parliament subservient to his wishes. But it made little difference whether the Parliament or the king controlled the state—in both cases the lives and activities of the people were subjected to bureaucratic controls. During most of this period the state controlled commodities, prices and wages. Certain rulers—such as Queen Elizabeth—made a practice of rewarding their court favorites by giving them an absolute monopoly on such necessities as salt, iron, oil, beer, and a vast number of other articles of commerce.

But in the eighteenth century the Anglo-American people demanded and progressively received more and more freedom at the hands of their rulers. Under the stimulus of freedom many new institutions came into being, the most important of which was the free market. And just as the free market grew and developed, to that extent the scope and responsibility of management did likewise. Just as the market became freer, to that extent management had a larger area in which to exercise its genius, initiative, ability, organizing capacity and managerial skill. Management by its very nature has always been inexorably tied to the free market. The one never could exist without the other.

The latter part of the eighteenth century found the American Colonists leading the fight for liberty. Many of the leaders among the Colonists had come to America in order to escape tyranny at home—but they found that England was exploiting the American Colonies to a point where manufacturing was well-nigh impossible. The Colonists were compelled to export their raw materials to England, where they were manufactured into finished products. They were also compelled to import goods manufactured in England. When to these prohibitions were added many vexatious taxes, such as that on sugar, on tea and

those imposed by affixing stamps, the ministers from their pulpits, the editors in their papers, and the lawyers in town meetings, stirred the people up by pointing out the injustices to which they were being subjected. The Revolution followed.

Then in 1787 came the Constitution of the United States—the greatest charter of liberty ever penned by the hand of man. The framers of this great document believed that the only useful purpose to be served by government was that of guaranteeing to its people life, liberty and the pursuit of happiness. They knew that political office conferred power on the recipient but that wisdom was acquired by quite a different process. They were determined that a man either elected or appointed to public office should not exercise that power to impair their freedom. Into that document they put positive limitations on all governmental power and specified that any power not expressly given to the Federal Government remained with the individual citizens and their local state governments. Freedom of the market place then became the accepted order.

Let me now discuss for a moment what constitutes a free market. My definition of a free market is one which comprehends the free exchange of goods at prices which the public is willing to pay, the free exchange of money with a value convertible into gold, free interest rates responsive only to the needs of finance and commerce, and freedom of the market place from intervention by government or any other outside force.

There never has been an absolutely free market in any country at any time in the world's history. But the conditions of a free market were approximately achieved right here in this country in the nineteenth century. During this period one invention followed after another and management was developed to a high point of efficiency. The products of these inventions were organized by management so that they might be mass-advertised, mass-marketed and then mass-produced. Economics, which is the science of the free market, was for the first time in history recognized as the only means of insuring a better and a finer way of life to the American people. The story of the development of the automobile and the oil industries is illustrative of what I have in mind.

Let me speculate with you as to what might have been the attitude of a Government Planning Board in 1900, if one had existed at that time, toward the automobile and petroleum industries. There were then in operation in this country some 8,000 motorcars, consuming 80,000 barrels of gasoline in a year. That is just about enough gasoline to keep all the cars of today in operation for 2½ minutes. Let us imagine Mr. Ford, with his great vision of the automobile's future, appearing before that Board and asking that in its program for the next two or three decades it provide a few billions of capital, along with the necessary labor and material, for manufacturing automobiles. The Board would have recognized in Mr. Ford a mild lunatic. They would have asked him where

he expected to get the gasoline for all those cars; and would have pointed out that neither the gasoline nor the crude oil from which to make it was anywhere in sight. And they would have rejected Mr. Ford's demands. A sophisticated public would have laughed at Ford when the Board set down genius as insanity and inventive ability as lunacy—and that would have ended all foolish talk about horseless carriages and flying machines.

But fortunately for the 30 million families in this country who derive pleasure and satisfaction from the operation of their cars, there was no such Board in 1900. And so Mr. Ford, not worrying about where his gasoline was coming from, went right ahead building more cars and better cars until presently he was turning out over a million cars a year.

Fortunately, too, for the oil industry, there was no such Board, for they went right ahead drilling more wells and deeper wells, and sometimes finding oil. They brought technology to their assistance in the form of geology and geophysics, and by their aid discovered new oil fields. They vastly improved refinery technology. And so the oil industry, doing each year those things which would have been impossible the year before, was always able to keep just a step ahead of the thirst for gasoline of those multiplying millions of automobiles.

You have all heard the old story that if you invented a better mousetrap the public would wear a path to your door. Now, I am going to shock you by stating that that story simply isn't true.

Intelligent management comprehends the coordination and application of all activities in the fields of research, physics, chemistry, psychology, sociology, accounting and statistics. It must direct research so that discoveries, when made, may be most conducive to the purposes for which the research was originally undertaken. When a new product is discovered, management provides the necessary engineering and design for a plant, which is then constructed with money obtained by management.

But management's biggest task still remains to be done, for the new product—like the mousetrap—cannot be disposed of until the people are educated as to its value. So a comprehensive marketing organization must be set up; and the products carried in stock at all important centers so as to meet the public demand for it.

Finally, a widespread educational advertising campaign, utilizing all media of communications, must be effected. If any one of these steps should be operated inefficiently, the success of the entire project is in jeopardy.

In 1787 it required 90 per cent of the entire population to produce the farm products which the people consumed. Today only one-sixth of our population live on farms. Science and management have provided the farmers with tools and know-how, so that this one-sixth of our population keeps the American people supplied with more and better food and raw materials than did 90 per cent of the population 160 years ago.

But what of the other five-sixths of our population today? Because

we have had a substantially free market in which rewards were commensurate with contributions, and because people were free to engage in any activity of their own choosing, most of these people entered some phase of industrial life. Management, constantly working out details for production and distribution, has guided our industrial development until the United States, with six per cent of the world's population, is consuming one-half of the world's industrial goods.

I have stressed the part freedom has played in the development of our industrial and economic life. I want now to stress the part that I believe freedom has played and must play in mangement. The most successful managements are those which make partners of their employees, and, respecting them as individuals, give them every opportunity to develop their initiative, skills and talents. This we have done in our Company and we never have had a strike.

But this freedom should obtain not only as between management and employees; it should motivate management in every phase of its activities. When a company engages in monopolistic practices, that company is suffering from bad management. When countries encourage and foster the creation of cartels, those countries are suffering both from bad industrial management and government. Freedom—freedom of the individual—is of the very essence of a successful economy—a successful country. Without it both men and countries die. The governmental wreckage that is strewn about us all over the world today is proof that this truth is fundamental.

Never were our frontiers so great as they are today. Scientific management and technology will supply the means for unlimited progress and invention in the future, provided they can operate in a free market.

The growth of invention resembles that of a tree. When planted in good soil, a tree throws out twigs; the twigs became branches, which in turn throw out more twigs—the process continuing in geometrical progression. So when a people operate in a free market, one invention becomes a source of many other inventions—the process continuing without limit.

This process is illustrated by the story of the Patent Office. About the middle of the last century it was proposed in Washington to erect a new building to house the Patent Office. A Congressional Committee called in Mr. Ellsworth, who was then United States Commissioner of Patents, to ask his advice. Commissioner Ellsworth counseled against too large or expensive a building, because, he stated, invention had just about reached its limit. He related the astounding advances that had been made in the mechanical arts in his lifetime, and predicted a cessation of endeavor in the field of invention—there just wasn't much of anything left to invent! At this point I made a little investigation of my own and found that up until Ellsworth's time there had been taken out in this country some 3,386 patents; and that since then over 2 million patents have been granted—just a little increase of some 60,000 per cent.

So much for that one government official who undoubtedly would have been a member of the Government Planning Board, if one had existed at that time. But had such a Planning Board existed, there would have been no need for a new Patent Office building, because the very existence of such a Government Planning Board would have destroyed the fertility of that economic soil which generates invention. And so the twigs of our patent tree would have shriveled up and died.

The problems of management are many and cover a wide field of human activity. The very fundamentals of a smooth-working economy presupposes the closest kind of cooperation among management, labor and capital. Let me illustrate by the legendary story of the old Arab.

This old Arab had three sons. Realizing that he had but a few years to live and being desirous of ascertaining which of the sons should succeed him as the head of the family, he called them together and told them that in a distant land were some scientists who had developed many amazing things. Then he instructed them to this effect: They were to journey together until they arrived at the distant land, where they should immediately separate. Each was to devote the following six months to a search for and acquisition of a present which would contribute most to the old Arab's welfare. They were then to meet while still in the distant land and journey home together.

At the appointed time they met and immediately displayed the presents which they had acquired. One had a magic carpet on which could be transported a number of people at an amazing speed; the second, a magic medicine which would cure any ill; the third, a magic glass through which could be seen what was transpiring in any home throughout the world.

So they turned the glass on their father's home and were dismayed to see that the old Arab was ill unto death. They then jumped onto the magic carpet, were quickly transported to their father's home, where the magic medicine was administered and the old Arab's life was saved.

Which of the three contributed most to the old Arab's welfare? Obviously the contribution of no one was of any value without that of each of the others. So it is with management, labor and capital. The contribution of no one of these is of value without that of the other two. Unless the American people can be brought to a realization of the dependence of each one of these groups on the other two, and that each is entitled to a reward in proportion to its contribution, our whole fabric of American life will be torn asunder.

Management has, during these last hundred years, been largely responsible for the creation of the greatest industrial machine of all time. In Washington today are many men in public office who use their political power to appropriate to themselves this great machine. To accomplish their purposes, they must first destroy the free market. To this end they have already succeeded in denying the right of the people to borrow and to loan money at rates of interest determined only by the

needs of commerce and finance. They have through inflation destroyed the integrity of our money; and they have in devious and diverse ways induced the Government to harass and interfere with the proper conduct of the market place. The only characteristic of the free market which remains today is the free exchange of goods at prices which the public is willing to pay—and even this has been under constant threat, attack and impairment.

Those in political power have on the one hand been appeasing more people by promising them more things, while on the other hand have been destroying the free market, the only way to assure more people more and better things. Among these promises are various types of phantom security. Now, management knows that the only security that can be effected is that which results from production and hard work. The great Danish philosopher, Kierkegaard, brings out this point in his parable of the wild duck.

With his mates this duck was flying in the springtime northward across Europe. During the flight he came down in a Danish barnyard where there were tame ducks. He ate of their corn and he liked it. He stayed for an hour—then for a day—then a week—then a month—and finally, because he relished the good fare and the safety of the barnyard, he stayed all summer. Then one autumn day when the flock of wild ducks were winging their way southward again, they passed over the barnyard and their mate heard their cry. His breast stirred with a strange thrill of joy and delight and, with a great flapping of wings, he rose in the air to join his old comrades in their flight—but he found his good fare had made him fat and his muscles so soft and flabby that he could rise no higher than the eaves of the barn. So he dropped back into the barnyard and said to himself, "Oh, well, my life is safe here and the food is good." But, alas, he was not safe from the man who fed him, for he later discovered that he was being fattened for the kill.

Inflation, which is the increase in the quantity of money, is management's most difficult problem—it is the world's greatest illusion. To the people inflation becomes an opiate. They think in terms of dollars, not in terms of plant, equipment and goods, and appear entirely oblivious to the fact that the purchasing power of their money is being destroyed.

To the Government, inflation is a device by which a large portion of the savings of the people can be siphoned off in the form of taxes. To management falls the responsibility to replace each year such of its plants and equipment as have worn out or become obsolete.

Government permits a charge for depreciation, but such charge is related to the original cost of the plant, whereas to replace a plant today costs on the average two and one-third times its original cost. My figures indicate that in the two years of greatest peacetime production—1947 and 1948—the real economic earnings of all American industry were less than one-fourth the monetary earnings which were reported. This meant that industry, after making modest payments to its stock-

holders and bond owners in the form of dividends and interest, had left insufficient money with which to replace its worn out and obsolete plant and equipment. The seriousness of this problem grows out of the fact that unless industry can effect as a result of its operations the means with which to reproduce itself, as and when its plant and equipment are worn out, our great industrial machine will gradually break down and fall into decay. For an appreciation of the misery which would follow such a disaster, look at Great Britain today!

There are men in our own country today clothed with political power who are diligently striving to socialize industry, medicine, education, labor and security. This is a totalitarian program and cannot be reconciled with the economy of the free market. There are many totalitarian regimes throughout the world today where the lives and activities of the people are planned by the state. Under such regimes, from the dictator down to the lowest serf, life is not worth the living, for every man lives in constant dread lest the state would throw him into prison or take his life. In contrast to this, during the last century you scientists, in an atmosphere of freedom, working through management, labor and capital, have wrought an indusrtial revolution far exceeding the fondest dreams of our Founding Fathers.

Our Founding Fathers were imbued with one great ideal—freedom for the individual. Anything which was in conflict with the proper exercise of that freedom, they believed to be wrong and therefore inimical to the interests of the people. Ever mindful of that ideal, they wrote the Constitution and the Bill of Rights. Under the Constitution and the Bill of Rights, freedom flourished. Little did our Founding Fathers realize that as a by-product of that freedom would come a fabulous development of material things. During these intervening years, people came more and more to regard material things as the greatest objective in life, while freedom for itself was pushed further and further into the background; until today they want more of these material things than it is possible for our present economy to provide—and so they would exchange their freedom for government handouts and subsidies. But when this is done, they will discover, too late, that they have lost both their freedom and the material things which they coveted.

The American people don't want socialism, but they don't understand the processes by which socialism is effected. To teach the American people a full appreciation of these processes, is the task of scientists and management.

Now, in conclusion, I appeal to you scientists to stand fast in defense of our American way of life and to keep our frontiers of freedom open and forever expanding; for I warn you that we are rapidly approaching the Point-of-No-Return on Freedom's Highway.

SPECIFIC-SURFACE DETERMINATIONS OF URANIUM DUSTS BY LOW-TEMPERATURE ADSORPTION OF ETHANE [1]

BY

KENNETH E. LAUTERBACH,[2] SIDNEY LASKIN,[2] AND LEONARD LEACH [2]

In the investigation of the physicochemical properties of uranium dusts, a study was made to determine the specific surface area of the uranium compounds used by this laboratory for toxicologic investigations.

Despite an extensive literature on the physicochemical properties of dusts, the relation of these properties to inhalation toxicity is not fully understood. Drinker and Hatch (1)[3] and Dalla Valle (2) have indicated that increased activity results from the size reduction of particles. Since this intensified physical and chemical activity can be most directly correlated with increased specific surface, it is reasonable that the toxicity of particulate material should be similarly related to solid surface. No such correlation can be shown, however, because of the lack of specific physical information for the commonly encountered dusts. In order to bridge this gap in our information, specific-surface measurements have been made on the uranium compounds that have undergone extensive toxicologic testing at the Rochester Atomic Energy Project (3).

A number of methods are available for determining the specific surface of finely divided solids and several reviews of all methods have been published (2, 4, 5). The low-temperature gas-adsorption method, notably the Brunauer, Emmett and Teller (BET) treatment (5), was most suited to our purpose for several reasons. The method did not depend upon the assumption of a regular geometrical structure for the material, and thus permitted an estimate of the surface irregularity and porosity. Also, low-temperature gas adsorption could not conceivably alter the solid surface present. The adsorption isotherm when plotted according to the BET equation required relatively few (3 or 4) data points for a single surface determination. Moreover, because our interest centered in a method for evaluating very small specific-surface values, the adaptation of the BET method by Wooten and Brown (6) was applied to the present problem.

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[1] This paper is based on work performed under contract with the United States Atomic Energy Commission at the University of Rochester Atomic Energy Project, Rochester, N. Y.

[2] Division of Pharmacology and Toxicology, Department of Radiation Biology, School of Medicine and Dentistry, The University of Rochester, Rochester, N. Y.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

THE BRUNAUER, EMMETT AND TELLER ADSORPTION EQUATION

The gas-adsorption method for the determination of the specific surface of solids depends upon the physical adsorption of gases near their boiling temperatures. A typical adsorption isotherm for nitrogen on an iron-alumina catalyst at −195.8° C. is replotted from the data of Brunauer and Emmett in Fig. 1 (upper curve). The graph is S-shaped with a central linear portion which represents the building up of multilayers of gas on the solid surface. Brunauer and Emmett con-

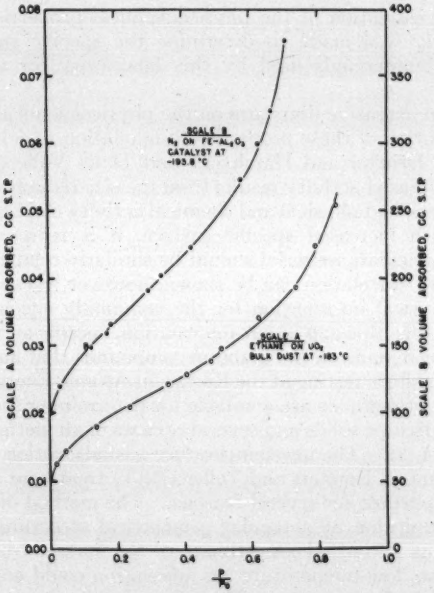

FIG. 1. Adsorption isotherms for ethane on uranium dioxide bulk dust at −183° C. and nitrogen on an iron-alumina catalyst at −195.8° C.

Vapor Pressures: Ethane at −183° C..... 0.01 mm.
Nitrogen at −195.8° C..... 760 mm.

cluded that the point at which a monolayer of gas covered the surface was *B*, the lowest pressure point at which the isotherm became a linear plot. The surface area of the solid sample was then simply the product of the number of molecules in this monolayer times the average area covered by each adsorbed molecule. The specific surface is the surface area per unit weight of the material.

In 1937 a mathematical treatment of the theory of adsorption by Brunauer, Emmett and Teller further substantiated the choice of *B* as

the point of monolayer adsorption. Their equation for adsorption of multimolecular layers of gases included both the Langmuir isotherm and the sigmoid isotherm as special cases and made it possible to obtain directly the volume of gas in the monolayer from a special plot of the adsorption data.

The complete equation given below has been referred to as the BET equation.

$$\frac{p}{V(p_0 - p)} = \frac{1}{V_m c} + \frac{(c-1)p}{V_m c p_0} \tag{1}$$

where

V = volume (S.T.P.) of gas adsorbed at pressure (p),
V_m = volume (S.T.P.) of gas required to form a monolayer,
p_0 = vapor pressure of the adsorbate at the temperature of adsorption, and
c = a constant related exponentially to the difference between the heat of liquefaction of the adsorbate and its heat of adsorption.

By plotting the adsorption data as $p/V(p_0 - p)$ *versus* p/p_0 a straight line is obtained over a limited pressure range. The slope and intercept of this line then determine V_m and c.

Emmett (5, 10) reported that the application of this method to several thousand finely divided substances using nitrogen at $-195.8°$ C. has shown a linear plot in the region between relative pressures of 0.05 and 0.35 except for a few materials. For general application to solids known to yield sigmoid isotherms, it is necessary to obtain only a few data points within this pressure range.

The absolute surface areas will depend also on the area occupied by each adsorbed molecule. Unfortunately, some arbitrariness is involved in selecting this value since molecular areas may be calculated from either the liquid or solid density of the adsorbate. Assuming close packing of the adsorbed molecules, the equation for the area per molecule is:

$$\text{Area per molecule} = 4(0.866)\left(\frac{M}{4\sqrt{2}N\rho}\right)^{\frac{2}{3}} \tag{2}$$

where M is the molecular weight of the adsorbate, N is Avogadro's number and ρ is the density of the liquefied or solidified gas. For nitrogen, Emmett (5) found the area per molecule calculated from the density of the liquefied adsorbate to be about 20 per cent greater than that from the solidified gas. Specific-surface values with nitrogen must therefore be regarded as uncertain by at least this amount although the method affords a way of comparing the relative areas of different adsorbents with an error of only a few per cent.

Recently, Wooten and Brown (6) extended the gas-adsorption method of Brunauer and Emmett to adsorbates of very low liquefac-

tion pressures in order to increase the sensitivity of the specific-surface measurements. Using butane and ethylene as adsorbates at respective temperatures of −116 and −183° C., they were able to evaluate surface areas as low as 25 cm.².[4] The adsorption isotherms closely resembled those of gases near their normal boiling temperatures, although condensation pressures were less than 0.1 mm.

EXPERIMENTAL METHOD

The uranium compounds used in this study consisted of samples of uranium dioxide, uranium trioxide, uranium peroxide, uranium tetrafluoride and uranyl nitrate hexahydrate. Two types of material were involved. The first consisted of micropulverized bulk compounds used dry for dust chamber dispersal in inhalation toxicity studies. The second type consisted of two sizes of uranium dioxide particles used in aqueous suspension for a series of particle-size toxicity studies (12). Methods for the preparation of these materials have been described (3, 12).

APPARATUS

The ethane adsorption apparatus (Fig. 2) was a modified form of that used by Wooten and Brown (6). Essentially it consisted of a sample tube, ethane and helium reservoirs, a McLeod gauge and a gas-purification train. Mercury seals were provided on all parts under high vacuum in order to eliminate stopcock leaks and contamination of the system with stopcock grease. The extra cut-off *E* was added to isolate the gas charging train from the adsorption system when adsorption determinations were being made. The apparatus was tested for leaks after complete degassing by closing the vacuum stopcock for 2 hr. If the pressure did not rise above 1×10^{-5} mm. Hg in this time, the system was considered air-tight.

The sample was protected from contamination by mercury vapor by an insert of gold foil fitted into the sample line after the last cut-off. Any traces of mercury which by-passed this trap were frozen out in a section of 2 mm. capillary above the sample and cooled by the liquid-oxygen bath.

The McLeod gauge was calibrated by filling with mercury. The bulb volume was 250 cm.³ with the capillary 0.5 mm. diameter providing a gauge that was sensitive to 1×10^{-5} mm. Hg. Gas-law corrections for ethane were negligible in the pressure range of the gauge (0.0005 to 0.05 mm. Hg).

Free space in the apparatus was determined by expanding a measured volume of helium from the McLeod gauge into the evacuated and cooled sample tube and measuring the resultant pressure. This volume

[4] Since completion of this study, our attention was brought to the work of Gaudin and Hukki (11) who used ethane as adsorbate at −183° C. (p_0 = 0.010 mm.) to determine the specific surface of crushed quartz samples.

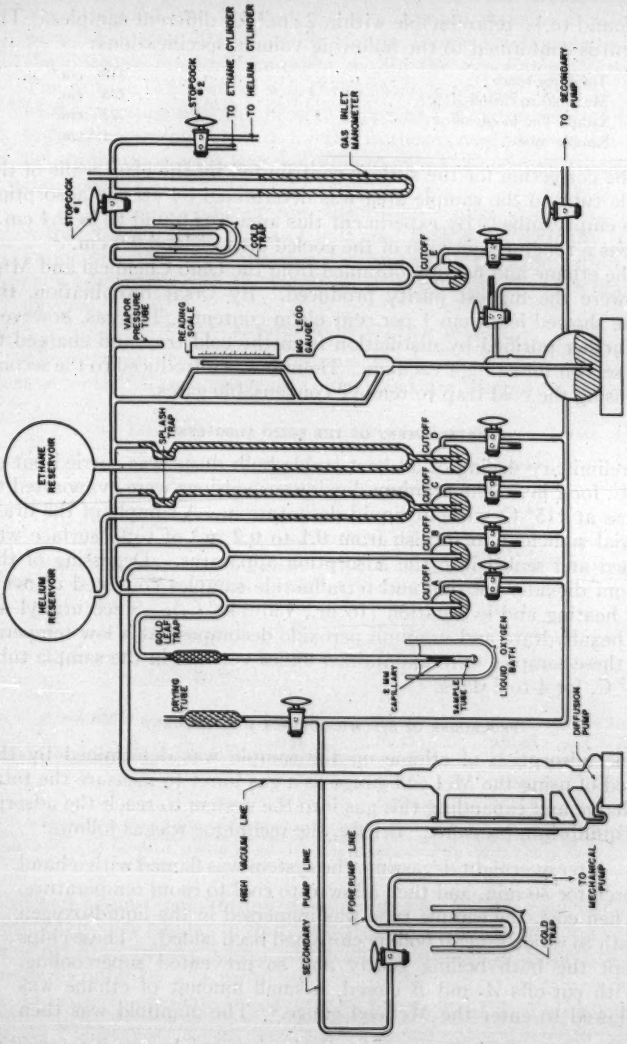

FIG. 2. Ethane-adsorption apparatus for the determination of surface area.

was found to be reproducible within 2 cm.^{3} for different samples. The apparatus conformed to the following volume specifications:

Total free space	318 cm.^{3}
Manifold to cut-off *A*	296 cm.^{3}
Sample line to cut-off *A*	22 cm.^{3}
Sample tube	1.4 cm.^{3}

The correction for the surface contributed by the glass walls of the sample tube to the sample area was determined by ethane adsorption on an empty tube. By experiment this area was found to be 9.4 cm.^{2}, whereas a rough calculation of the cooled area yielded 9.9 cm.^{2}.[5]

The ethane and helium, obtained from the Ohio Chemical and Mfg. Co., were the highest purity produced. By Orsat bromination, the ethane showed less than 1 per cent olefin content. The gas, however, was further purified by distillation from the cold trap and charged to the reservoir under high vacuum. Helium was introduced to the second bulb using the cold trap to remove condensable gases.

PRETREATMENT OF THE SOLID ADSORBENT

Preliminary drying of all heat-stable bulk dusts was carried out at 105° C. for 2 hr. The uranium dioxide suspensions were evaporated to dryness at 115° C. without liquid decantation. A sample of the dried material sufficient to furnish from 0.1 to 0.2 m.^{2} of total surface was weighed and sealed into the adsorption apparatus. Degassing of the uranium dioxide, trioxide and tetrafluoride samples consisted of overnight heating and evacuation (16 hr., 130–140° C.). Since uranyl nitrate hexahydrate and uranium peroxide decompose at a low temperature, these samples were maintained under vacuum in the sample tube at 25° C. for 4 to 5 days.

PROCEDURE OF SPECIFIC-SURFACE MEASUREMENT

The adsorption of ethane on the sample was determined by the method of using the McLeod gauge as a gas buret to measure the total gas charge and expanding this gas into the system to reach the adsorption equilibrium pressure. Briefly, the technique was as follows:

> After overnight degassing, the system was flamed with a hand torch for 30 min. and then allowed to cool to room temperature. When cool, the sample tube was immersed in the liquid-oxygen bath to which several boiling chips had been added. These chips kept the bath boiling gently and so prevented supercooling. With cut-offs *A* and *B* closed, a small amount of ethane was allowed to enter the McLeod gauge.[6] The manifold was then

[5] This exceptionally close agreement is probably coincidental; however, it was gratifying to know that the method is reasonably accurate at such a low surface value.

[6] Small amounts of either helium or ethane were conveniently introduced into the manifold by first lowering the mercury in the proper cut-off until the level in the left column dropped

evacuated and after closing the vacuum cut-off the measured gas was expanded into the system. Equilibrium pressure readings were taken after 20 min. and in 10-min. intervals following, until the pressure remained constant within the experimental error. The sample tube cut-off was then closed, more gas added to the McLeod gauge and the process repeated. This method of following increasing pressure for the isotherm was adopted for all determinations, as it was desirable to avoid the complications of the desorption technique.

In this way, four or five adsorption values were obtained within the pressure range of 0.05 and 0.35 p/p_0. Finally, the vapor pressure of ethane was determined at the completion of each adsorption with the vapor-pressure tube immersed in liquid oxygen.

ETHANE ADSORPTION ON URANIUM DIOXIDE

The data for a typical adsorption of ethane on uranium dioxide bulk dust at −183° C. are given in Table I and are plotted as gas volume

TABLE I.—*Adsorption of Ethane at −183° C. on Bulk Uranium Dioxide.*

	Adsorption Pressure, μ Hg	p/p_0	Adsorbed Volume, cc., S.T.P.	$\frac{p}{V(p_0 - p)}$
Part I.[a]				
p_0 = 0.00925 mm. Hg	0.14	0.015	0.0124	1.23
	3.26	0.352	0.0241	22.5
	3.74	0.404	0.0258	26.2
	4.72	0.510	0.0295	35.4
	5.76	0.622	0.0338	—
	6.76	0.731	0.0380	—
	7.35	0.795	0.0439	—
	7.91	0.855	0.0499	—
Part II.				
p_0 = 0.00979 mm. Hg	0.41	0.042	0.0144	3.01
	1.34	0.137	0.0180	8.82
	2.32	0.237	0.0207	15.0
	3.47	0.354	0.0240	22.9
	4.65	0.475	0.0275	32.9

[a] In order to obtain a sufficient number of adsorption points, the determination was performed in two parts on different days. Between measurements, the sample was heated *in vacuo* overnight. On the second day, an appreciable rise in the barometric pressure caused the boiling temperature of the oxygen bath to rise giving a higher vapor pressure for ethane.

adsorbed against equilibrium pressure in Fig. 2 (lower curve). This isotherm is very similar to that for nitrogen on an iron-alumina catalyst at −195.8° C. (upper curve), although the respective adsorbate vapor pressures differ by a factor of 10^5.

In order to evaluate the volume of the monolayer (V_m), the ethane

past the Y intersection. Surface tension held the mercury column in the right side of the cut-off stationary. The valve was then reversed immediately, isolating a small bubble of gas in the left line and forcing it into the splash trap.

isotherm was replotted according to the BET relationship in Fig. 3. The points in this graph fall on a straight line from 0.05 to nearly 0.35 p/p_0. Since the slope of this line is $c - 1/V_m c$ and the intercept is $1/V_m c$, V_m can be calculated as:

$$1/(\text{slope} + \text{intercept}).$$

The absolute value of the surface area is obtained from an estimate of the average coverage per molecule. For ethane at $-183°$ C., the area per molecule was calculated as 20.50×10^{-16} cm.^{2}. This is based on a solid density value of 0.609 g./cm.^{3} determined from X-ray data

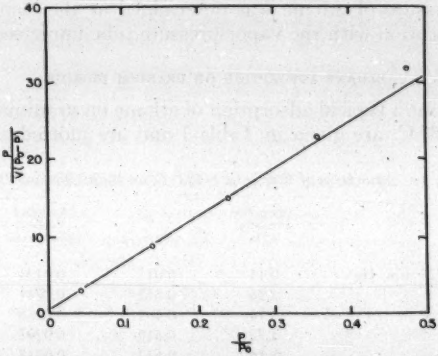

FIG. 3. Plot of $p/V(p_0 - p)$ *vs.* p/p_0 for the ethane adsorption on uranium dioxide shown in Fig. 1.

Sample size	0.219 g.
Specific surface	0.41 m.^{2}/g.

(13). Referring to V_m (Eq. 1) the area present per cm.^{3} of gas at S.T.P. is:

$$\frac{(20.50 \times 10^{-16})(6.02 \times 10^{23})}{(22.4 \times 10^3)(1 \times 10^4)} \text{ or } 5.51 \text{ m.}^2/\text{cm.}^3.$$

Finally, the specific surface of x grams of a sample will be:

$$\frac{5.51 V_m - S}{x} \text{ in terms of m.}^2/\text{g.},$$

where S is the area of the glass walls of the sample tube.

The value of V_m determined from the plot in Fig. 3 for the ethane adsorption on uranium dioxide has been found to be 0.0164 cm.^{3} at S.T.P. and the corrected surface of the sample 0.41 m.^{2}/g. This specific surface for uranium dioxide was one of four independent determinations on the same material. The other values were 0.43, 0.44 and 0.45 m.^{2}/g. and the average deviation was 3.5 per cent from the mean of 0.43 m.^{2}/g.

PARTICLE-SIZE, SPECIFIC-SURFACE RELATIONSHIP

In order to correlate the experimentally determined specific surfaces with those obtained from particle-size measurements, the following relations, based on the methods of Perrott and Kinney (14) and Green (15), were developed.

If the particle is assumed to be of some uniform shape, *viz.*, spherical and nonporous, the experimentally determined specific surface should agree with that calculated by the relationship:

$$\text{Specific surface} = \frac{\text{Total surface of the particles}}{\text{Total weight of the particles}} = \frac{N(\pi D_s^2)}{N\frac{(\pi D_w^3)}{6}\rho} = \frac{6D_s^2}{\rho D_w^3} \tag{3}$$

where

N = the number of particles,
D_w = the median particle diameter determined from the size frequency data weighted in terms of mass,
ρ = the density of the particle, and
D_s = the median particle diameter determined from the size frequency data weighted in terms of surface.

For a cubical particle, the final relationship is the same as that for spherical particles.

In most cases, the experimental data indicate a much larger surface area than that calculated from particle-size measurements. This deviation is probably caused by the divergence of the particle from the nonporous or regular condition. Since porosity may increase particle surface to a greater extent than it affects particle weight or diameter, the relationship must be revised to include combined shape-porosity factors. Thus where K_s is the surface shape factor and K_w is the weight shape factor then:

$$\text{Specific surface} = \frac{6D_s^2}{\rho D_w^3} \times \frac{K_s}{K_w}. \tag{4}$$

The ratio of these shape factors may be defined as a porosity factor, P. It is then a logical assumption that the difference between the two methods of determining specific surface is the result of the porosity. Therefore,

$$P = \frac{K_s}{K_w} = \frac{\text{Specific surface by gas adsorption}}{\text{Specific surface by particle-size measurement}}. \tag{5}$$

When the porosity factor is unity, the material can be considered as nonporous. In applying the particle-size data to the above relationship ρ, the density of the average particle, is assumed to be equal to the density of the original material. The surface and mass median particle sizes are calculated from original size measurements by methods similar to those described previously (3).

TEST OF THE GAS-ADSORPTION METHOD BY STANDARDIZED GLASS BEADS

As an independent proof of the accuracy of the ethane-adsorption method for determining the specific surface of finely divided materials, several adsorptions were made on sized glass beads.[7] Because Emmett and others had obtained experimental specific-surface values that exceeded theoretical by 40 per cent and assumed the deviation to result from etching of the glass surface during cleaning (4, 5), the beads used for this study were untreated after the firing process by the manufacturer. Microscopic examination of bead samples showed only negligible amounts of contaminating material. By direct observation of 3000 beads, only 2.4 per cent of the beads were found to be nonspherical. Specific-surface values were calculated from microscopic measurements in the manner previously discussed.

The average size mass median for the samples was 102 micra while in terms of surface the average median was calculated as 100 micra. The specific surface calculated from these measurements yielded a value of 0.023 m.²/g. An average of the specific surfaces determined by ethane adsorption on the same samples gave a value of 0.025 or a positive difference of 9 per cent. The porosity value of the beads according to Eq. 5 was calculated to be 1.1. This approaches the experimental error of the method and provides what is considered to be a very good agreement between experimental and theoretical values.

RESULTS WITH URANIUM COMPOUNDS

The values for specific surface and porosity of the uranium compounds investigated are shown in Table II. For all the micropulverized bulk dusts the specific-surface values determined experimentally were relatively low. With the exception of uranium peroxide for which a value of 5.74 m.²/g. was obtained, these values were of the order of several tenths of a square meter per gram, ranging from 0.38 m.²/g. for the trioxide to 0.71 m.²/g. for the tetrafluoride. The corresponding specific surfaces calculated from direct microscopic measurements showed the same relative range of values for all compounds. For the dioxide, trioxide and nitrate, the porosity factors ranged from 1.1 to 1.9, indicating relatively nonporous materials. The porosity factor of 3.7 for uranium tetrafluoride indicated some surface irregularity, whereas a

[7] The glass beads were of the type used in egg timers and were obtained from Seed Filter Co., 47 East Merrick Road, Fairport, N. Y.

significant increase was shown by the value of 11.7 obtained for the peroxide sample.

Samples of UO_2, finely ground, were separated from aqueous suspensions by elutriation. Gram for gram, the uranium dioxide samples labeled "Fine Suspensions" in Table II showed a ten-fold larger specific surface (4.23 m.²/g.) than that for the "Coarse Suspensions" (0.47 m.²/g.). The corresponding specific surface values calculated from microscopic measurements were significantly lower: 0.299 and 0.069 m.²/g., respectively. Inasmuch as the microscopic values showed only

TABLE II.—*Specific-Surface and Porosity Values of Uranium Compounds of Different Particle Sizes.*

Type of Sample	Compound	Particle-Size Medians		Specific Surface		Porosity
		Mass, micra	Surface, micra	Ethane Adsorption, m.²/g.	Microscope Measurements, m.²/g.	
Bulk Dusts (Micropulverized)	Uranium Dioxide	1.83	1.62	0.43	0.24	1.8
	Uranium Trioxide	1.62	1.36	0.38	0.36	1.1
	Uranium Peroxide	2.55	2.29	5.74	0.49	11.7
	Uranium Tetrafluoride	3.13	2.49	0.71	0.19	3.7
	Uranyl Nitrate Hexahydrate	4.09	3.44	0.69	0.37	1.9
Suspensions [a]						
Coarse	Uranium Dioxide	6.26	5.56	0.47	0.069	6.8
Fine	Uranium Dioxide	1.25	1.03	4.23	0.299	14.1

[a] These dusts were obtained by sedimentation and decantation; samples were dried for surface-area determinations.

a five-fold difference which could be directly correlated with a reduction in particle size, the comparative factor of 2 between the methods was attributed to a change in the physical structure of the material or a leaching of the surface associated with the process of suspending the materials (12). The factor of 2 is also shown in the calculated porosity values which were 14.1 for the fine suspension and 6.8 for the coarse suspension.

Some evidence of a change in physical structure or of a leaching effect is indicated in a comparison of the results obtained with these suspensions and those obtained for the bulk micropulverized samples of the same material (Table II), for which an average porosity value of 1.8 was obtained. Apparently the exposure to water in the process of elutriation increases the surface area. These results are similar to the increased surface values associated with the washing of sized glass beads as reported by Emmett and others (4, 5).

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AN EXTENSION OF A THEORY OF MAGNETIC AMPLIFIERS [1]

BY

ROBERT T. BEYER [2] AND MING-YI WEI [2]

ABSTRACT

A theory of operation of certain magnetic amplifiers, previously presented, is extended, in the case of secondaries with infinite a-c. impedance to be valid for large input currents. The second harmonic output is shown to deviate from linearity (with d-c. input) and pass through a maximum as the direct current is increased. Some experimental evidence is given to support the calculations. The influence of a third harmonic term in the primary current (not considered in the simple theory) is also investigated. It is shown that for small input currents no serious error is introduced by neglecting the higher harmonic.

1. INTRODUCTION

A simple theory of operation for certain magnetic amplifiers or nonlinear transformers has been presented in a previous paper (1).[3] In such magnetic amplifiers, the primary winding is excited by a current of audio frequency, of sufficient amplitude to drive the ferromagnetic core into the region of saturation. The addition of a direct current in any secondary winding on the transformer will produce an asymmetry in the hysteresis loop. This results in the appearance of even harmonic components in the voltages across the various windings. These magnetic amplifiers have had successful application in various types of computing circuits, particularly those involving addition, integration and differentiation (2).

In the work I. cited in reference 1, an analysis was made of the voltages which should appear across the various windings on a toroidal core transformer made of a ferromagnetic material. It was assumed that the current in the primary winding was sinusoidal, and large enough to drive the core into the saturation region in each cycle. It was further assumed that the current in all secondary windings was sufficiently small that a linearized form of the mesh equations could be readily set up and solved.

Among the limitations of this theory is the fact that the primary voltage, rather than the primary current, is sinusoidal, or nearly so. This means that the primary current will contain odd harmonics, particularly the third.

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[1] A portion of the work reported in this paper is part of a thesis submitted by the latter author to the Graduate School of Brown University for the Degree of Master of Science.

[2] Professor and Graduate Student, respectively, Physics Department, Brown University, Providence, R. I.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

A second limitation is that the theory applies only to small d-c. inputs. Since many of the applications depend critically upon the linearity of the relation between input d-c. and the amplitude of the second harmonic output voltage, the behavior of the magnetic amplifiers for large d-c. inputs is also of interest.

The purpose of the research reported here was to extend the theory of **I.** to take account of these variations in the operating conditions. Some experimental results are also given. More detailed experiments, designed to check the validity of the entire theory, are now in progress, and will be reported at a later date.

2. INITIAL ASSUMPTIONS AND SIMPLE THEORY

As a point of departure, a brief summary of the simple theory will be given. Let us consider a ferromagnetic core, of length l and cross-sectional area A, with W sets of windings.[4] It is assumed that all secondaries are a-c. open-circuited, that the relation $B = F(H)$ is known, that there is no flux leakage, and that B is uniform over any section of the core. Hysteresis effects are neglected, and only the steady state is considered.

If in the j-th mesh, associated with the j-th winding, there is a generator with e.m.f. E_j and linear impedance $Z_j(p)$ $\left(\text{where } p \text{ is the operator } \frac{d}{dt}\right)$, the set of W mesh equations can be written as

$$E_j = Z_j(p)I_j + N_j g(I_1, I_2, \cdots, I_W) \sum_{k=1}^{W} N_k p I_k, \tag{1}$$

where

$$g(I_1, I_2, \cdots, I_W) = \frac{0.4\pi A}{l} \times 10^{-8} \left(\frac{dB}{dH}\right)_{H=M/l}.$$

Here B and H are in electromagnetic units while E, Z, I and g are in practical units. The dimensions of g are those of a reactance.

Under the assumption that the primary current is much larger than any of the secondary currents, it is possible to expand the function $g(I_1, I_2, \cdots, I_W)$ in a Taylor series about the point P: I_1, 0, 0, $\cdots$, 0. Equation 1 can then be written as

$$E_j = Z_j(p)I_j + N_j N_1 g(I_1)_P p I_1 + N_j g(I_1)_P \sum_{k=2}^{W} N_k p I_k + N_j (p I_1) \sum_{k=2}^{W} \left(\frac{\partial g}{\partial I_k}\right)_P I_k + \text{higher order terms}, \tag{2}$$

where

$$g(I_1)_P = g(I_1, 0, 0, \cdots, 0).$$

[4] A table of the symbols used in this paper is included in Appendix B.

The expression assumed for $F(H)$ is:

$$B = \frac{2S}{\pi} \tan^{-1}\left(\frac{\pi\mu_0}{2S} H\right) = \frac{2S}{\pi} \tan^{-1}(\alpha H) \tag{3}$$

where S = saturation value of B and μ_0 is the initial permeability.

With the added assumption that the primary current is sinusoidal, it is possible to expand the various terms on the right side of Eq. 2 in Fourier series involving harmonics of the driving frequency. The components resulting from the term $N_jN_1g(I_1)_P pI_1$ are odd harmonics of the fundamental frequency, resulting entirely from the effect of the non-linear characteristic on the primary current. The components resulting from

$$N_j(pI_1) \sum_{k=2}^{W} \left(\frac{\partial g}{\partial I_k}\right)_P I_k$$

are even harmonics resulting from the appearance of direct currents in the secondaries. Thus the amplitudes of the even harmonic voltages are linear functions of the input direct currents, a result which is characteristic of such magnetic amplifiers (3, 4). The term

$$N_j g(I_1)_P \sum_{k=2}^{W} N_k pI_k$$

appears only if the secondary currents have a-c. components, which is not the case under the assumptions of this analysis.

3. ANALYSIS FOR LARGE DIRECT CURRENTS

The analysis for large direct currents parallels the treatment outlined above, except that the function $g(I_1, I_2, \cdots, I_W)$ is expanded about the point P: $I_1, I_2, 0, \cdots, 0$. With this change, and with the assumption that $I_1, I_2 \gg I_k$, $k \neq 1, 2$, Eq. 2 becomes

$$E_j = Z_j(p)I_j + N_1N_j g(I_1, I_2, 0, \cdots, 0)pI_1. \tag{4}$$

The magnetic field intensity is now given by

$$H = \frac{0.4\pi}{l}[N_1I_1 + N_2I_2] \tag{5}$$

or, if we let $I_1 = I_{10} \sin \omega t$, $I_2 = I_{20}$,

$$H = \frac{0.4\pi}{l}[N_1I_{10} \sin \omega t + N_2I_{20}]. \tag{5a}$$

We now expand $N_1N_jg(I_1, I_2)_P pI_1$ in a Fourier series:

$$N_1N_jg(I_1, I_2)_P pI_1 = \tfrac{1}{2}G_0 + \sum_{n=1}^{\infty} (G_n \cos n\omega t + H_n \sin n\omega t). \tag{6}$$

The coefficients G_n and H_n are given by

$$\begin{matrix} G_n \\ H_n \end{matrix} = \frac{2\gamma}{T} \int_{-(T/2)}^{T/2} \begin{matrix} \cos \\ \sin \end{matrix} n\omega t \frac{\cos \omega t dt}{1 + \beta^2(\sin \omega t + \psi)^2} \tag{7}$$

where

$$\beta = \frac{0.4\pi\alpha N_1 I_{10}}{l}$$

$$\psi = \frac{N_2 I_{20}}{N_1 I_{10}}$$

$$\gamma = \frac{0.4\pi A \mu_0}{l} \times 10^{-8}.$$

The solutions of these integrals are given in Appendix A. It follows from these solutions that

$$G_n = \frac{\gamma}{\beta}\left[\exp\left\{in \sin^{-1}\left(\frac{i}{\beta} + \psi\right)\right\} + \exp\left\{in \sin^{-1}\left(\frac{i}{\beta} - \psi\right)\right\}\right] \quad (n \text{ odd})$$

$$= 0 \quad (n \text{ even or zero}) \tag{8}$$

$$H_n = \frac{i\gamma}{\beta}\left[\exp\left\{in \sin^{-1}\left(\frac{i}{\beta} + \psi\right)\right\} - \exp\left\{in \sin^{-1}\left(\frac{i}{\beta} - \psi\right)\right\}\right] \quad (n \text{ even})$$

$$= 0 \quad (n \text{ odd}).$$

If ψ is made very small, the expression for G_n reduces to

$$G_n \cong \frac{2\gamma}{\beta} \exp\left(-n \sinh^{-1}\frac{1}{\beta}\right), \quad (n \text{ odd}) \tag{9}$$

which is the solution obtained in I. Under these circumstances, the expression for H_n reduces to that for the even harmonics. Thus the entire solution is contained in the single Fourier expansion rather than the two previously required. The expressions for G_n and H_n are, however, in a more complicated form than those obtained in I. The advantage of Eq. 8 is that the specific behavior can be determined for large ψ, that is, for large input current.

As an example, H_2 has been calculated numerically for $\beta = 2.00$. This value of β corresponds, in an approximate fashion, to the actual value which prevails in the experimental arrangements. The results of these calculations are shown in Fig. 1. The solid line represents the values of $\frac{1}{\gamma} H_2$ as a function of ψ. This curve is a measure of the ampli-

tude of the second harmonic voltage output for various direct current inputs. The departure from linearity is represented by the broken line, which is a plot of $H_2/\psi \left(\frac{dH}{d\psi}\right)_{\psi=0}$.

It can be seen from these curves that the relation of H_2 *vs.* ψ gradually departs from linearity as ψ becomes appreciable. For example,

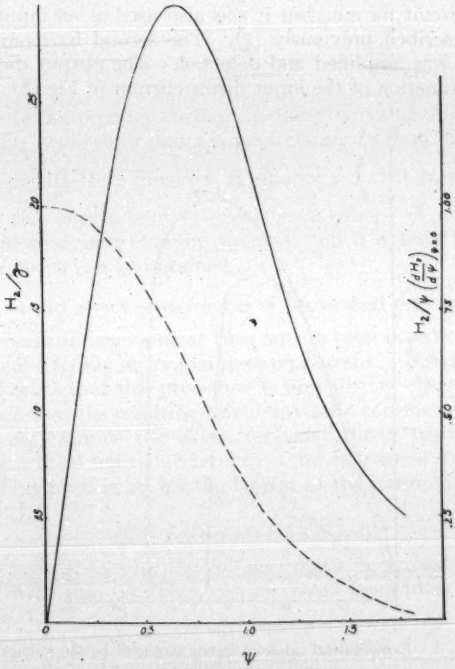

FIG. 1. Calculated values of second harmonic output voltage. Solid line represents output voltage *vs.* input current; broken line represents sensitivity *vs.* input current.

the value of H_2/ψ remains within 1 per cent of the limiting value (as $\psi \to 0$) as long as ψ is less than 0.1, that is, as long as

$$I_2 < 0.1 \frac{N_1}{N_2} I_{10}. \tag{10}$$

It also follows that a maximum second harmonic output is reached

for $\psi = 0.7$, that is, for

$$I_2 = 0.7 \frac{N_1}{N_2} I_{10}. \tag{11}$$

A rough experimental check was made on these values. An excitation voltage of 1000 cycles was applied to a 200-turn primary of a non-linear transformer with a molybdenum permalloy core. The transformer had a single secondary of 200 turns. The secondary was essentially open-circuit for a.c., but it was also used as an input winding, a technique described previously (2). The second harmonic output of this winding was amplified and detected. The output d-c. voltage is plotted as a function of the input direct current in Fig. 2. A different

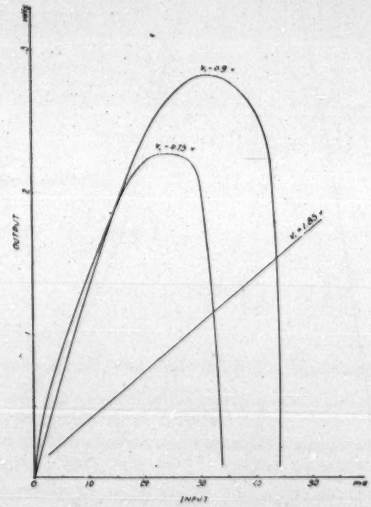

FIG. 2. Experimental values of second harmonic output voltage *vs.* input current at various primary voltages.

primary voltage was used in each case; for curve 1, 0.75 volts rms.; for curve 2, 0.90 volts rms.; and for curve 3, 1.85 volts rms.

The rms. value of the primary current was measured in each case and the value of I_{10} was computed, *under the assumption that the primary current is sinusoidal*. It is realized that in the actual case the primary current is not sinusoidal, but the method serves as a rough measure of the validity of the analysis. The effect of higher frequency components will be considered later.

The values of the input direct current at which maximum output

voltage was obtained are given in the fifth column, Table I. The corresponding theoretical values, calculated from Eq. 11, are listed in the fourth column. It can be seen that these experimental results are in essential agreement with the analysis. It must be emphasized that the techniques employed here are very rough. For example, a change

TABLE I.

Curve (Fig. 2)	Primary Voltage, volts rms.	I_{1s}, ma.	$0.7\, I_{1s}\frac{N_1}{N_2}$, ma.	(I_2) max. output, ma.
1	0.75	35	24	25
2	0.90	43	30	32
3	1.85	240	170	>50

in the value of the primary current modifies the value of β. However, a glance at Eq. 8 will show that a larger β (larger I_{10}) will make the curve of H_2 *vs.* ψ depart from linearity at smaller ψ $\left(\text{that is, smaller } \frac{I_{20}}{I_{10}}\right)$. Thus these two effects tend to cancel each other. A more elaborate experimental analysis is now in progress, and it is hoped that a more satisfactory check can be obtained.

4. THE EFFECT OF HARMONICS IN THE PRIMARY CURRENT

The theoretical development thus far has been based entirely on the assumption of a purely sinusoidal primary current. It has been pointed out several times that this procedure is unrealistic. Its main justification has been that the resulting calculations are considerably simplified. We shall now examine the effect, for small direct current inputs, of introducing a third harmonic term into the expression for the primary current. This term is by far the largest of the harmonics in ordinary operation. Let

$$I_1 = I_{10}(\sin \omega t - \delta \sin 3\omega t). \tag{12}$$

The Fourier coefficients for the voltage appearing across an open circuit secondary, with zero d-c. bias, can be found from integrals similar to those of Eq. 7:

$$\begin{matrix} G_n \\ H_n \end{matrix} = \frac{2\gamma}{T}\int_{-(T/2)}^{T/2} \begin{matrix} \cos \\ \sin \end{matrix} n\omega t \, \frac{\cos \omega t dt}{1 + \beta^2(\sin \omega t - \delta \sin 3\omega t)^2}. \tag{13}$$

Since the integrand for H_n is an odd function of t, $H_n = 0$ for all n. Furthermore, the integrand of G_n is anti-symmetric, for even n, about $t = \pm \frac{T}{4}$, so that G_n vanishes in such cases.

Since in practice the voltage across the secondary is very largely of the fundamental voltage, one must in effect vary the parameters β and δ until the values of G_1, G_3, etc., fit the experimental conditions. To do this, G_1 and G_3 have been evaluated numerically for various β and δ. The results of these numerical integrations are shown in Figs. 3 and 4.

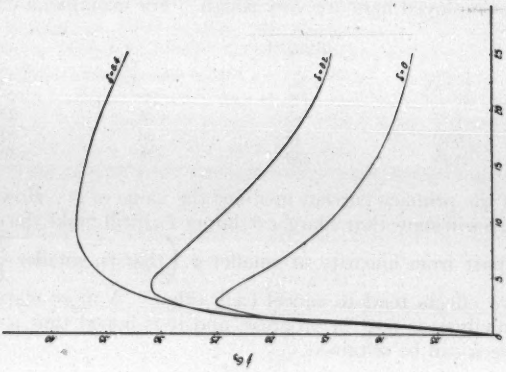

FIG. 4. Calculated values of third harmonic output voltage as a function of primary current.

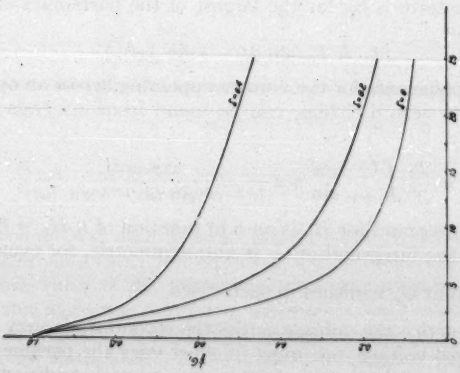

FIG. 3. Calculated values of fundamental output voltage as a function of primary current.

In Fig. 3, the values of $\frac{1}{\gamma} G_1$ (which is a measure of the fundamental output) are plotted as a function of β for $\delta = 0$, 0.20 and 0.40. The values at $\delta = 0.20$ and 0.40 are obtained from the numerical integration, while those at $\delta = 0$ were obtained from the closed form solution obtained in I. Similar graphs of $\frac{1}{\gamma} G_3$ (proportional to the third harmonic voltage) are shown in Fig. 4.

These graphs may be interpreted in the following way. The inclusion of a third harmonic in the primary current reduces the value of β below that which would be computed under the assumption of a pure sinusoidal current of the fundamental frequency. From the graphs it can be seen that a decrease in the value for β increases the ratio of G_1/G_3, which is in closer agreement with experimental conditions.

For example, if $\beta = 2.0$ and $\delta = 0$, the calculated ratio of the fundamental amplitude to that of the third harmonic is 3 to 1, while for $\beta = 1.0$ and $\delta = 0.2$, the ratio is 8 to 1.

The above treatment does not furnish us with the even harmonics. To find these we re-write Eq. 2. For simplicity, we restrict ourselves to two windings, and let $I_2 = I_{20}$, a constant. Then the voltage across the secondary is given by

$$E_2 = R_2 I_{20} + N_2 N_1 g(I_1)_P p I_1 + N_2 (p I_1) \left(\frac{\partial g}{\partial I_2} \right)_P I_{20}. \tag{14}$$

Here we shall be interested only in the last term, which we shall designate by e_2. From the work in I. it can be shown that

$$\left(\frac{\partial g}{\partial I_2} \right)_P = \frac{N_2}{N_1} \left(\frac{\partial g}{\partial I_1} \right)_P. \tag{15}$$

Also,

$$(p I_1) \left(\frac{\partial g}{\partial I_1} \right)_P = \frac{\partial g_P}{\partial I_1} \frac{\partial I_1}{\partial t} = \frac{\partial g_P}{\partial t} \tag{16}$$

so that

$$e_2 = \frac{N_2^2}{N_1} \frac{\partial g_P}{\partial t} I_{20}. \tag{17}$$

It is then only necessary to expand g_P in a Fourier series and take the time derivative in order to find the harmonic components of e_2. Now

$$g_P = \frac{\gamma}{1 + \beta^2 (\sin \omega t - \delta \sin 3\omega t)^2} \tag{18}$$

$$= \tfrac{1}{2} K_0 + \sum_{n=1}^{\infty} (K_n \cos n\omega t + L_n \sin n\omega t),$$

where

$$\begin{matrix} K_n \\ L_n \end{matrix} = \frac{2j}{T} \int_{-(T/2)}^{T/2} \begin{matrix} \cos \\ \sin \end{matrix} n\omega t \frac{dt}{1 + \beta^2(\sin \omega t - \delta \sin 3\omega t)^2}. \tag{19}$$

It can be seen that $L_n = 0$ for all n and $K_n = 0$ for odd n. (The value of K_0 is not important, since the term disappears in the next step.) The voltage e_2 is then

$$e_2 = -\frac{N_2^2}{N_1} I_{20}\omega[2K_2 \cos 2\omega t + 4K_4 \cos 4\omega t + \cdots]. \tag{20}$$

Numerical integration of K_2 has been carried out for $\delta = 0.2$ and 0.4 as a function of β. The results of these integrations are shown in Fig. 5, together with the curve for $\delta = 0$, which was obtained in I.

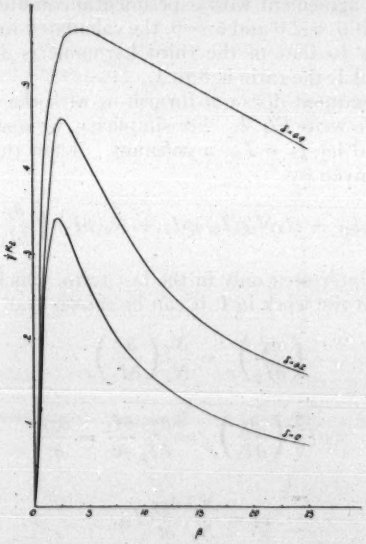

FIG. 5. Calculated values of second harmonic output voltage as a function of primary current.

It can be seen from these graphs that the use of a smaller β than 2.00 for any δ decreases the computed sensitivity of the cores to the production of the second harmonic. However, this is compensated by an inincrease in sensitivity, for given β, with larger values of δ. It may therefore be concluded that the analysis on the basis of a purely sinusoidal current is more accurate than would have otherwise appeared.

5. SUMMARY

The analysis of this paper eliminates two of the restrictions on a theory of magnetic amplifiers, namely, small input currents and sinusoidal primary current. This has been done only for the case of secondaries which are effectively open-circuited. One may draw the general conclusion that the presence of harmonics in the primary current does not seriously affect the conclusions of the simple theory. On the other hand, the use of large direct currents requires the use of the more accurate treatment.

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(2) H. S. SACK, R. T. BEYER, G. H. MILLER AND J. W. TRISCHKA, "Special Magnetic Amplifiers and Their Use in Computing Circuits," *Proc. I.R.E.*, Vol. 35, p. 1375 (1947). Gives specific details concerning these applications.

(3) T. EPSTEIN, German Patent No. 149761, August, 1902.

(4) J. ZENNECK, "A Contribution to the Theory of Magnetic Frequency Changers," *Proc. I.R.E.*, Vol. 8, p. 468 (1920).

APPENDIX A

The integrals involved in Eq. 7 are of the form

$$I_{\pm} = \int_{-\pi}^{\pi} \frac{e^{\pm iKZ} dZ}{1 + \beta^2(\sin Z + \psi)^2}, \tag{21}$$

and the evaluation is similar to that given in I. for the case of $\delta = 0$. The poles of the integral are at

$$Z = \sin^{-1}\left(\frac{i}{\beta} - \psi\right) \pm n\pi \qquad (n \text{ even})$$

$$Z = \sin^{-1}\left(\frac{i}{\beta} + \psi\right) \pm n\pi \qquad (n \text{ odd})$$

The location of the poles and the corresponding contour for $K > 0$ are shown in Fig. 6. Evaluation of the two residues

$$R_0\left[Z = \sin^{-1}\left(\frac{i}{\beta} - \psi\right)\right] \quad \text{and} \quad R_1\left[Z = \sin^{-1}\left(\frac{i}{\beta} + \psi\right) - \pi\right]$$

gives

$$R_1 = \frac{(-1)^K \exp\left\{iK \sin^{-1}\left(\frac{i}{\beta} + \psi\right)\right\}}{2i\beta\sqrt{1 - \left(\frac{i}{\beta} + \psi\right)^2}} \tag{22}$$

while

$$R_0 = \frac{\exp\left\{iK \sin^{-1}\left(\frac{i}{\beta} - \psi\right)\right\}}{2i\beta\sqrt{1 - \left(\frac{i}{\beta} - \psi\right)^2}} \tag{23}$$

and

$$I_+ = 2\pi i(R_0 + R_1).$$

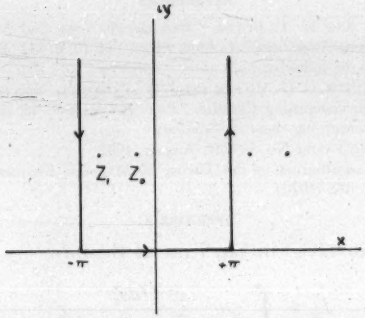

FIG. 6. Contour and poles for integration of Eq. 21.

For negative K the contour must be taken in the negative half plane. This eventually yields the solution

$$I_- = \frac{\pi}{\beta}\left[\frac{\exp\left\{-iK \sin^{-1}\left(-\frac{i}{\beta} - \psi\right)\right\}}{\sqrt{1 - \left(\frac{i}{\beta} + \psi\right)^2}} + \frac{(-1)^K \exp\left\{-iK \sin^{-1}\left(-\frac{i}{\beta} + \psi\right)\right\}}{\sqrt{1 - \left(-\frac{i}{\beta} + \psi\right)^2}}\right]. \tag{24}$$

The particular integrals involved in Eq. 7 can be evaluated by combinations of I_+ and I_-.

APPENDIX B

Table of Symbols

B	flux density
H	magnetic field intensity
M	magnetomotive force
S	saturation value of flux density
A	cross sectional area of core
l	length of core
W	number of sets of windings on core
N_j	number of turns on j-th winding
I_j	current in j-th winding
E_j	emf. in j-th mesh
$Z_j(p)$	linear impedance in j-th mesh
p	d/dt
ω	angular frequency of driving oscillator
μ_0	initial permeability of core
I_{10}	amplitude of primary current of fundamental frequency
I_{30}	amplitude of primary current at third harmonic
I_{20}	direct current in secondary
$g(I_1, I_2, \cdots, I_W)$	$\dfrac{0.4\pi A}{l} \times 10^{-8} \left(\dfrac{dB}{dH}\right)_{H=M/l}$
$g(I_1)_P$	$g_P(I_1, 0, 0, \cdots, 0)$
$g(I_1, I_2)_P$	$g_P(I_1, I_2, 0, 0, \cdots, 0)$
ψ	$\dfrac{N_2 I_{20}}{N_1 I_{10}}$
e_2	even harmonic components of voltage across secondary winding
G_n, H_n	Fourier coefficients of expansion for $N_1 N_j g(I_1, I_2)_P p I_1$
K_n, L_n	Fourier coefficients of expansion for $g(I_1)_P$
β	$\dfrac{0.4\pi\alpha N_1 I_{10}}{l}$

Samuel S. Fels.—The Franklin Institute has lost a valued officer and friend in the death of Samuel S. Fels, on June 24, at Temple University Hospital in Philadelphia.

Mr. Fels first affiliated himself with the Institute in 1909, when he became a member. Elected to the Board of Managers in 1930 and to a Vice-Presidency in 1934, Mr. Fels faithfully served on the governing body of The Franklin Institute until his death. The Board of Managers recognized his services in 1943 by electing him to Honorary Life membership in the Institute.

Mr. Fels' interest in science was demonstrated in the many research projects undertaken by the Samuel S. Fels Fund, and in the gift of the Fels Planetarium to The Franklin Institute. The Planetarium was opened to the public in 1933, and on that occasion Mr. Fels said, in part:

"After all, the heavens are ours to love and to study if our cities only have the ingenuity to annex them. They have a special meaning for us in times such as these. The earth under our feet is fixed and we tend to fashion our habits of thought on it. The heavens constantly change, but there is order in their movements. We can draw confidence from them in addressing ourselves to the problems and promises of our changing world.

"The Benjamin Franklin Memorial seemed to be the natural channel through which such an opportunity might reach the people of Philadelphia; so I joined with its trustees and as a result, here it is before you. If the satisfaction that has come to me in my efforts to bring it to you are equalled by the pleasure and profit that others receive from it, my object will have been accomplished."

Mr. Fels' generosity in presenting the Planetarium to the Institute has permitted many thousands of young and old to be, in Mr. Fels' own words, "lifted out of themselves."

For his many humanitarian acts, he was given the Philadelphia Award in 1948. His many activities were recognized in the form of honorary degrees from three institutions of higher learning: the University of Pennsylvania granted him a Doctor of Laws degree in 1937; Antioch College awarded him a Doctor of Science degree in 1947; and Temple University gave him a Doctor of Science degree in 1949.

Samuel S. Fels was born February 16, 1860, to the wife of the village postmaster and storekeeper of Yanceyville, North Carolina. The family moved first to Baltimore and then to Philadelphia during the Reconstruction following the Civil War, where they founded the business which has since borne their name. From 1881 to 1914 he was a partner in this business—Fels & Co., makers of the famous Fels Naphtha Soap—and in 1914 became President. He held that position until his death. In 1890 he married Jennie M. May of New Haven, Conn., who died in 1934. The couple had no children.

In addition to his work at the Fels plant and at the Institute, he was founder of the Crime Prevention Association; co-founder of the Big Brothers Association; a founder-member of the Committee of 70; and a founder and director of the Bureau of Municipal Research.

ZIRCONIUM AS A MATERIAL FOR FRACTIONAL WEIGHTS

BY

WILLIAM M. THORNTON, JR.[1] AND EDWARD S. HAUBER, S.J.[1]

INTRODUCTION

In a typical box of analytical weights, the fractions of the gram, from 500 mg. down to, and including, 50 mg., are composed of platinum; whereas the remaining weights, 20 mg. to 1 mg., inclusive, are of aluminum. However, there has been a tendency in recent years to substitute other materials for platinum. Gold alloys (preferably, 22-carat gold) have been used to a considerable extent, but these auric compositions have not proved to be entirely satisfactory (6).[2] On the contrary, sheet-metal weights of tantalum are very serviceable; it would seem that their constancy has been adequately established (9).

Zirconium exhibits greater similarity to tantalum than to either platinum or aluminum. It is a fairly hard metal, yet it is ductile when pure. In like manner, it possesses numerous other physical properties that are desirable. Zirconium is markedly inert toward active chemicals, though, like tantalum, it succumbs to hydrofluoric acid (3, 10). Finally, in an interesting newspaper article, White (12) ventures the prophecy that high-grade zirconium will be much more plentiful, and much less expensive, in the future than it has been in the past.

In a previous communication (9), an attempt was made to summarize the requirements that should be met by reliable reference weights. On examining these criteria anew, the present writers find that there are two respects in which zirconium warrants serious attention.

First, it appears now that the melting point of zirconium is not inordinately high. De Boer (3) gives 1857° C. Waggaman and Gee (10) put down 1860° C. (?), thus indicating that the present value for this constant should be taken with some qualification. But if these data are even approximately correct, the successful fusing of zirconium ought not to be an insurmountable undertaking, seeing that the transition temperature is only some 84° C. above that of platinum. And it is well known that a thoroughly melted metal is much more likely to be non-porous—that is, to attain the highest known density—than one which has been consolidated by some process akin to forging (*powder metallurgy*, for example).

Secondly, the density of zirconium is such as to make the corrective term to be employed in converting "apparent mass *versus* brass" to "true mass" advantageously small (cf. 7). Weiss and Neumann (11) report

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

6.4 g. per ml. at 18° C. as the density of their reguline zirconium. De Boer (3), however, gives the slightly higher value 6.53, but he fails to specify the temperature. It seems best to accept the result of De Boer—chiefly on the ground that his material must have been very nearly pure. Accordingly, a quantity of zirconium having a "true mass" of 1 gram would evince a "weight in air" of 0.999,959 g.; in other words, the buoyancy correction would be +0.041 mg. The analogous adjustment in the case of platinum is −0.087 mg.; for tantalum, −0.070 mg.; and for aluminum, +0.302 mg.

DESCRIPTION OF THE ZIRCONIUM WEIGHTS AND PRELIMINARY TESTS THEREON

The material, from which the fractional weights under consideration were constructed, was a sizable sheet of metallic zirconium donated by Mr. E. C. Pitzer, of the Foote Mineral Company (Philadelphia). One square centimeter of this sheet weighed approximately 0.077 g.

Four small weights were produced from the zirconium under the supervision of Mr. Gordon S. Clayson, Vice-President and General Manager, Wm. Ainsworth and Sons, Inc. (Denver). They were received in Baltimore on August 21, 1946. The work of construction and adjustment was performed with all necessary nicety. The finished articles present a very attractive appearance.

TABLE I.—*Comparison of Σ(100 mg.)Zr with Standard (No. 6239).*

Date	Apparent Mass *vs.* Brass, mg.	Atmospheric Conditions
Sept. 11, 1946	100.002	...
Sept. 11, 1946	100.002	[a]
Oct. 23, 1946	100.002	...
Nov. 5, 1946	100.002	[b]
Feb. 11, 1948	100.002	[c]
March 26, 1948	100.002	[d]

[a] 27.8° C., 761.1 mm., and 56.5 per cent relative humidity.
[b] 23.6° C., 762.7 mm., and 51 per cent relative humidity.
[c] 22.6° C., 766.4 mm., and 30 per cent relative humidity.
[d] 25.4° C., 757.6 mm., and 31 per cent relative humidity.

The designations of the several weights are as follows: 50 mg., 20 mg., 20 mg. (one dot), and 10 mg. The superficial dimensions of the 5-cg. piece in zirconium are slightly greater than those of the corresponding Ainsworth weight in platinum; whereas the remaining members of the zirconic series are somewhat less in area than the analogous weights in aluminum.

The weights under test were cleaned with the help of redistilled alcohol; whereupon they were allowed two days in which to become apparently dry at the temperature prevailing in the balance room (about 32° C.).

Between the second and the third determination of Σ(100 mg.)Zr—see Table I—the four weights were placed in a desiccator, containing anhydrous calcium chloride, and permitted to remain there for 24 hr.

Reweighing, at this juncture, revealed no diminution of mass. It appears, therefore, that the zirconium surfaces had reached a state of equilibrium with respect to the surrounding atmosphere.

A platinum decigram (No. 6239), which had been carried through two rather elaborate group calibrations, was chosen as the standard of reference in the present work. The value of this decigram, in terms of the standards of the United States, is 100.004 mg. ("weight in air") or 99.996 mg. ("true mass").

All weighings pertaining to the present investigation were performed with a moderately sensitive assay balance. As the functioning of this instrument is to be described in another publication, no further elucidation is necessary (9a).

The zirconium weights, taken together, were compared with the standard by the method of substitution, and the difference was noted:

$$+ \Sigma(100 \text{ mg.})\text{Zr} - (\text{S } 100 \text{ mg.}) = - 0.002 \text{ mg.} \qquad (1)$$

Therefore, $\Sigma(100 \text{ mg.})\text{Zr} = 100.002$ mg. ("weight in air").

DETERMINATION OF Σ (100 mg.)Zr

The comparison of Σ(100 mg.)Zr with the fiducial weight (No. 6239), in order to obtain the aggregate value of all the zirconium pieces, has been described briefly above. This determination is most important, for it is the step upon which the entire standardization (2) primarily depends; hence the desirability of repeating the procedure several times. Accordingly, six such comparisons were made, the total lapse of time between the first and the last being practically 18½ months.

In addition to the foregoing, it is well to follow the advice of Pienkowsky (8), who directs that such composite values as that represented by Σ(100 mg.)Zr should be determined both before and after the intercomparisons (see Tables II and III). In this manner, not only does the

TABLE II.—*First Calibration of Zirconium Weights.*[a]

Designation	Apparent Mass vs. Brass, mg.	True Mass, mg.
(50 mg.)Zr	49.997	49.999
(20 mg.)$_1$Zr	20.003	20.004
(20 mg.)$_2$Zr	20.000	20.001
(10 mg.)Zr	10.002	10.002

[a] Test completed: October 14, 1946.

TABLE III.—*Second Calibration of Zirconium Weights.*[a]

Designation	Apparent Mass vs. Brass, mg.	True Mass, mg.
(50 mg.)Zr	49.997	49.999
(20 mg.)$_1$Zr	20.003	20.004
(20 mg.)$_2$Zr	20.000	20.001
(10 mg.)Zr	10.002	10.002

[a] Test completed: March 26, 1948.

second result serve to verify the first in a general way, but, presumably, it would also reveal any appreciable change in the weights brought about by the numerous manipulations.

INTERCOMPARISON OF THE ZIRCONIUM WEIGHTS

For ascertaining the values of the individual weights composed of zirconium one of the simpler calibration schemes of Benoit (2) was followed. In this procedure, there are eight observational equations in relation to five unknown quantities. Therefore, the operator may calculate the most probable values for the several weights by the method of least squares (4); and the residuals reveal the consistency with which the comparisons have been made. Inasmuch as four weights only constitute the zirconium group, the expedient was adopted of introducing a centigram of aluminum in place of Σ(10 mg.)Zr. The correction for this extra weight had been carefully determined; hence it served as a control upon the preceding experimental work (cf. Pienkowsky (8)).

The observational equations and their numerical equivalents, as obtained in the first calibration, follow:

$$+ (50\text{ mg.})\text{Zr} - \Sigma(50\text{ mg.})\text{Zr} = a_1 = -0.008\text{ mg.} \quad (2)$$

$$+ (50\text{ mg.})\text{Zr} - (20\text{ mg.})_1\text{Zr} - (20\text{ mg.})_2\text{Zr} - (\text{S }10\text{ mg.}) = a_2 = -0.006\text{ mg.} \quad (3)$$

$$+ (20\text{ mg.})_1\text{Zr} - (20\text{ mg.})_2\text{Zr} + (10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = a_3 = +0.004\text{ mg.} \quad (4)$$

$$+ (20\text{ mg.})_1\text{Zr} - (20\text{ mg.})_2\text{Zr} - (10\text{ mg.})\text{Zr} + (\text{S }10\text{ mg.}) = a_4 = +0.002\text{ mg.} \quad (5)$$

$$+ (20\text{ mg.})_1\text{Zr} - (20\text{ mg.})_2\text{Zr} = a_5 = +0.002\text{ mg.} \quad (6)$$

$$+ (20\text{ mg.})_1\text{Zr} - (10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = a_6 = +0.002\text{ mg.} \quad (7)$$

$$+ (20\text{ mg.})_2\text{Zr} - (10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = a_7 = -0.002\text{ mg.} \quad (8)$$

$$+ (10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = a_8 = +0.002\text{ mg.} \quad (9)$$

Equations 2 to 9, inclusive, give rise to the subjoined normal equations:

$$+ 2(50\text{ mg.})\text{Zr} - 2(20\text{ mg.})_1\text{Zr} - 2(20\text{ mg.})_2\text{Zr} - (10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = A_1 \quad (10)$$

$$- 2(50\text{ mg.})\text{Zr} + 6(20\text{ mg.})_1\text{Zr} - (20\text{ mg.})_2\text{Zr} = A_2 \quad (11)$$

$$- 2(50\text{ mg.})\text{Zr} - (20\text{ mg.})_1\text{Zr} + 6(20\text{ mg.})_2\text{Zr} = A_3 \quad (12)$$

$$- (50\text{ mg.})\text{Zr} + 6(10\text{ mg.})\text{Zr} - (\text{S }10\text{ mg.}) = A_4 \quad (13)$$

$$- (50\text{ mg.})\text{Zr} - (10\text{ mg.})\text{Zr} + 6(\text{S }10\text{ mg.}) = A_5 \quad (14)$$

in which

$$A_1 = +a_1 + a_2 = -0.014 \text{ mg.}$$
$$A_2 = -a_1 - a_2 + a_3 + a_4 + a_5 + a_6 = +0.024 \text{ mg.}$$
$$A_3 = -a_1 - a_2 - a_3 - a_4 - a_5 + a_7 = +0.004 \text{ mg.}$$
$$A_4 = -a_1 + a_3 - a_4 - a_6 - a_7 + a_8 = +0.012 \text{ mg.}$$
$$A_5 = -a_2 - a_3 + a_4 - a_6 - a_7 - a_8 = +0.002 \text{ mg.}$$

(See Baten (1) and Kohlrausch (5) for the rule to be followed in deriving the normal equations, and for a brief exposition of the theory of least squares.)

On solving Eqs. 10 to 14, inclusive, and simplifying the resulting expressions with the aid of Benoit's identity ($5A_1 + 2A_2 + 2A_3 + A_4 + A_5 = 0$), one obtains the computation formulae.

In these formulae, M denotes Σ(100 mg.)Zr.

$$(50 \text{ mg.})\text{Zr} = \tfrac{1}{2}M + \tfrac{1}{28}(7A_1 - A_4 + A_5) = 49.997 \text{ mg.} \quad (15)$$

$$(20 \text{ mg.})_1\text{Zr} = \tfrac{1}{5}M + \tfrac{1}{35}(A_1 + 5A_2 - A_4) = 20.003 \text{ mg.} \quad (16)$$

$$(20 \text{ mg.})_2\text{Zr} = \tfrac{1}{5}M + \tfrac{1}{35}(A_1 + 5A_3 - A_4) = 20.000 \text{ mg.} \quad (17)$$

$$(10 \text{ mg.})\text{Zr} = \tfrac{1}{10}M + \tfrac{1}{140}(7A_1 + 23A_4 + 5A_5) = 10.002 \text{ mg.} \quad (18)$$

$$(\text{S } 10 \text{ mg.}) = \tfrac{1}{10}M + \tfrac{1}{140}(7A_1 + 3A_4 + 25A_5) = 10.000 \text{ mg.} \quad (19)$$

RESIDUALS AND CHECK DETERMINATIONS

First Calibration

$$R_1 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (1)$$
$$R_2 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (2)$$
$$R_3 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (3)$$
$$R_4 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (4)$$
$$R_5 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (5)$$
$$R_6 \text{ (obs. } - \text{ calc.)} = -0.001 \text{ mg.} \quad (6)$$
$$R_7 \text{ (obs. } - \text{ calc.)} = 0.000 \text{ mg.} \quad (7)$$
$$R_8 \text{ (obs. } - \text{ calc.)} = -0.001 \text{ mg.} \quad (8)$$
$$R_9 \text{ (obs. } - \text{ calc.)} = +0.001 \text{ mg.} \quad (9)$$

Second Calibration (Second Members Only of Observational Equations and the Corresponding Residuals)

$$a_1 = -0.008 \text{ mg.} \qquad R_{20} = 0.000 \text{ mg.} \quad (20)$$
$$a_2 = -0.006 \text{ mg.} \qquad R_{21} = 0.000 \text{ mg.} \quad (21)$$
$$a_3 = +0.004 \text{ mg.} \qquad R_{22} = 0.000 \text{ mg.} \quad (22)$$
$$a_4 = 0.000 \text{ mg.} \qquad R_{23} = 0.000 \text{ mg.} \quad (23)$$

$$a_5 = +0.002 \text{ mg.} \qquad R_{24} = 0.000 \text{ mg.} \tag{24}$$

$$a_6 = +0.002 \text{ mg.} \qquad R_{25} = +0.001 \text{ mg.} \tag{25}$$

$$a_7 = -0.002 \text{ mg.} \qquad R_{26} = -0.001 \text{ mg.} \tag{26}$$

$$a_8 = +0.002 \text{ mg.} \qquad R_{27} = 0.000 \text{ mg.} \tag{27}$$

Check Determinations

Previously known value for (S 10 mg.) = 10.000 mg. (mean of closely agreeing results obtained in six calibrations). Value found (in both first and second intercomparison of zirconium weights) = 10.000 mg. Difference = 0.000 mg.

CONCLUSIONS

Apart from the concordant numbers for the composite value, Σ(100 mg.)Zr, in Table I, which imply a satisfactory permanence on the part of the fractional weights of zirconium, the calibration data for the individual pieces in Tables II and III, respectively, are in agreement to the nearest microgram. Moreover, the smallness of the residuals, and the gratifying control determinations on the standard centigram, (S 10 mg.), justify one in believing that the experimental results are acceptably accurate.

As soon as zirconium of the requisite purity becomes sufficiently abundant, and procurable at a price that is not excessive, it is reasonable to suppose that the fractional weights in a box can be fashioned from this metal with some decided advantages—certainly, in so far as the decigram and centigram groups are concerned. It might still be well to employ aluminum for the milligram group, in order to maintain a suitable relation between area and thickness.

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AN ELECTRONIC INTEGRAL-TRANSFORM COMPUTER AND THE PRACTICAL SOLUTION OF INTEGRAL EQUATIONS

BY

HENRY WALLMAN [1]

1. INTRODUCTION

One of the most frequently used concepts of mathematics, physics, and engineering is that of the integral transform: given the real function $f(t)$ of one real variable, and the real function $K(x, t)$ of two real variables, the function $F(x)$ defined by

$$F(x) = \int_a^b K(x, t)f(t)dt, \tag{1}$$

is called the *integral transform of* $f(t)$ *with kernel* $K(x, t)$. This paper describes a proposed instrument for rapid electronic computation (about 1/10 second) of such integral transforms, with graphical presentation of the results on a cathode-ray tube screen, and then discusses the mathematical implications of such an instrument.

A feature of this method is that only very minor changes are needed to deal with non-linear integral transforms (Sec. 6).

Included among the special cases of (1) are the Fourier transform (Sec. 7), the Hilbert transform (Sec. 9), and the convolution integral (Sec. 10). The proposed instrument makes it possible to expand, in 1/10 second, an arbitrary function in Laguerre polynomials, Legendre polynomials, Tschebyscheff polynomials, etc. (Sec. 8).

The ability to compute integral transforms rapidly permits the solution of integral equations (Secs. 11–16), especially in view of the ready adaptability of the present method to successive iteration (Sec. 5). A special case of an integral equation is that of a family of simultaneous linear algebraic equations; such families are discussed in Sec. 12.

Integral equations are of great importance in solving boundary-value problems of ordinary and partial differential equations. An example given in Sec. 15 leads to an integral equation which, although non-linear, is capable of being solved very elegantly with the proposed instrument.

A second example (Sec. 16), of greater generality, illustrates the Green function method of solving partial differential equations with specified boundary values. The problem is the classical Dirichlet boundary-value potential problem in the plane: with the proposed instrument it will be necessary only to draw simple graphs of (*a*) the

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parametric equations of the given arbitrary contour (assumed convex), and (*b*) the given arbitrary boundary value function. In a few minutes a solution to the Dirichlet problem will be obtained.

I. THE INSTRUMENT

2. SIMPLIFYING ASSUMPTIONS

It is assumed that for the purposes of computation it suffices (*a*) to take a and b finite, and (*b*) to have x range over a finite number of discrete steps, for example $x = 0, 1, 2, \cdots, 99$. (Note that the variable t remains continuous.)

We therefore write

$$F(x) = \int_a^b K(x, t)f(t)dt, \qquad x = 0, 1, \cdots, 99 \tag{2}$$

and desire to present on a cathode-ray tube screen a graph of $F(x)$ against x, as shown in Fig. 1.

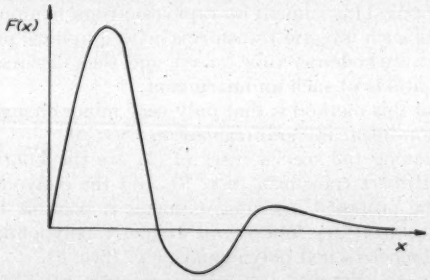


FIG. 1. Integral transform of $f(t)$ with kernel $K(x, t)$, as presented on cathode-ray tube.

3. PRINCIPLE OF THE METHOD

The method proceeds in steps, with one step for each x-value, $0, 1, \cdots, 99$, and each step yielding one point of the graph of $F(x)$.

Step n.

Set $x = n$. With t varying continuously between a and b, multiply the function $f(t)$ by the function $K(n, t)$. Form the integral of this product between the limits $t = a$ and $t = b$. This integral is $F(n)$ and is employed as the vertical deflection of a cathode-ray tube spot whose horizontal deflection is $x = n$ units.

As x increases from 0 to 99 a sequence of dots thus traces out 100 points of the graph of $F(x)$. If the range $t = a$ to $t = b$ corresponds to a time interval of 1/1000 second, the complete graph of 100 points of $F(x)$ will be drawn in 1/10 second; the entire process is then repeated.

To avoid flicker a cathode-ray tube screen is employed having long persistence of light emission.

4. APPARATUS

The required apparatus is shown in block diagram in Fig. 2 and consists of: (*a*) a high-speed two-variable function-generator, (*b*) a high-speed one-variable function-generator, (*c*) a multiplier, (*d*) an averager, and (*e*) a cathode-ray tube with sweeps. Of these components only (*a*) has not yet been built, but its development appears entirely reasonable,

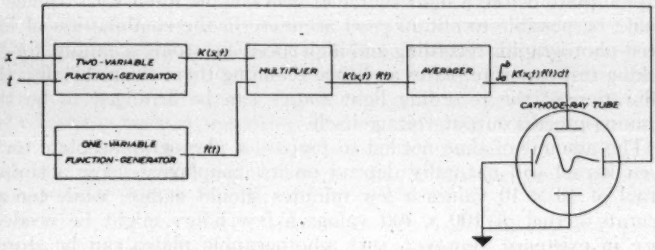


FIG. 2. Block diagram of integral-transform computer. The variable t sweeps linearly between 0 and 1, in 1/1000 second, say, and then repeats. Throughout a given t-sweep x is constant, and increases one unit, from 0 through 99, for each complete t-sweep.

being in accordance with standard television technique. Components (*b*) and (*c*) have already been developed,[2] and components (*d*) and (*e*) are routine.

(*a*) *High-Speed Two-Variable Function-Generator*

This device, which is the technical crux of the transform-computer, is one that for each of a set of discrete values of x converts the arbitrary real function $K(x, t)$ into a proportional time-dependent voltage. This end can be achieved with a photographic plate divided into vertical strips, having one strip for each value of x, and with optical opacity at position (x, t) of the x-th strip proportional to $K(x, t)$. A plate, or slide, of this sort could well be called a 'photograph' of the kernel $K(x, t)$ (compare Fig. 4). The conversion from the variable opacity of the photographic plate to a variable electrical voltage is to be effected by the familiar high-speed television scanning process, making use of an iconoscope or image dissector. If there are 100 values of x, that is, 100 'lines' in the photograph of $K(x, t)$, and if 1/1000 second is allotted

[2] Solutions to the problems of constructing components (*b*) and (*c*) are described in a Massachusetts Institute of Technology doctor's thesis (1948) on "An Electronic Differential Analyzer," by Alan B. Macnee, *Proc. I.R.E.*, Vol. 37, pp, 1315–1324 (1949), carried out under the author's supervision. Especially noteworthy is Dr. Macnee's "crossed-fields" electron-beam multiplier.

to each line, the entire function $K(x, t)$ will be scanned in 1/10 second, neglecting retrace time.

A photographic opacity range of about 100/1 or 200/1 is not difficult to obtain. To accommodate both positive and negative values of $K(x, t)$, a specified photographic opacity level is made to correspond to zero, with greater opacities representing positive values and lower opacities negative values.

With regard to accuracy the following can be said: the function-generator for a given $K(x, t)$ can be prepared by slowly moving a photographic plate before a light source of controllable intensity. Hence it should be possible to obtain good accuracy in the combination of low speed photographic recording and high speed electronic scanning, for by making test exposure plates and then scanning them electronically, the calibration of the recording light source can be arranged to be the scanning-process output voltage itself.

The amount of time needed to prepare a photographic plate for a given kernel will naturally depend on its complexity. For a simple kernel of 10×10 values a few minutes should suffice, while for an accurate kernel of 100×100 values a few hours might be needed. Once in existence, however, such photographic plates can be stored against all future use of the same kernel, thus building up a library of useful kernels.

(*b*) *High-Speed One-Variable Function-Generator*

This is a device converting an arbitrary real function $f(t)$ of a single real variable into a similar time-dependent voltage. The arbitrary function is given as a graph projected onto the face of a cathode-ray tube, and the conversion to a time-function is carried out by means of a curve-following cathode-ray beam, the time required to traverse the graph being 1/1000 second.

The one-variable function-generator requires as input data only a graph. Hence a few seconds suffice to construct a one-variable function, in contrast to the many minutes, or even a few hours, required for a two-variable function. Fortunately, in any one problem involving integral-transforms the kernel $K(x, t)$ is usually held fixed while $f(t)$ is varied; with the present method, after one has obtained the transform of a function $f(t)$, only a few seconds will be needed to set up and obtain the transform with the same kernel of a different $f(t)$.

(*c*) *Multiplier*

This is a device that takes two time-dependent voltages and yields a time-dependent voltage proportional to their product.

(*d*) *Averager*

This is a device yielding a voltage proportional to the average of a given time-dependent voltage.

(*e*) *Cathode-Ray Tube*

The horizontal deflection is proportional to x and the vertical deflection to $F(x)$, as is standard.

5. ITERATED TRANSFORMS

It is often desirable (see Sec. 13) to be able to iterate transforms, that is to take the transform of a function $f(t)$ with the kernel $K(x, t)$, thereby obtaining a function $F(x)$, and then to take the transform, with the same kernel, of $F(x)$. A somewhat similar operation is discussed in Sec. 7.2, in which one has to take the sine transform of the cosine transform of a given function.

The persistence of light emission from a cathode-ray tube screen makes the process of iterating transforms possible. It will be necessary to project the trace representing $F(x)$ on the output cathode-ray tube, which, as already stated, has a persistent screen, onto the face of the cathode-ray tube employed in the one-variable function-generator (component (*b*) of Fig. 2).

In effect this procedure employs the output cathode-ray tube screen as a simple storage device, and in that way permits the graph of $F(x)$ to be written down (repetitively) in 1/10 second, but yet to be read off in 1/1000 second. If some such 'time-compression' scheme were not employed there would be a hundred-fold increase in time duration with each successive iteration of a transform; this would be prohibitive for the use of the instrument in the iterative solution of integral equations.

6. NON-LINEAR TRANSFORMS

Difficult problems sometimes arise involving non-linear integral transforms, of the types

$$\int K(x, t)k(t, f(t))dt, \tag{3a},$$

$$\int K(x, t)k(x, f(t))dt, \tag{3b},$$

$$\int K(x, t)f(k(x, t))dt, \tag{3c},$$

where $K(x, t)$ is a kernel, $k(\xi, \eta)$ a given function of two variables, and the function being transformed is $f(t)$. In any one problem K and k will ordinarily be held fixed, while f will be varied; for the purposes of computation it is then desirable to be able to obtain very quickly, for given K and k, the transforms of different functions $f(t)$. This can be done by extension of the techniques so far described.

Some examples, for different k, illustrate the scope of (3). For

case (3*a*):[3]

$$\int K(x, t)(1 + e^{f(t)} \sin t)dt, \qquad \int K(x, t) \sin f(t)dt, \qquad \int K(x, t)(f(t))^3 dt, \tag{4a}$$

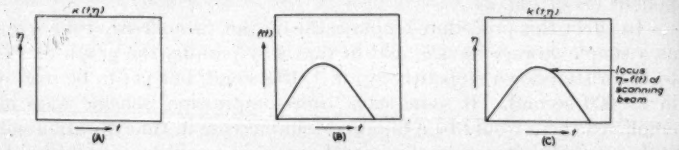
for case (3*b*):

$$\int K(x, t)e^{2^{f(t)}}dt, \qquad \int K(x, t)(f(t))^x dt, \tag{4b}$$

and for case (3*c*):

$$\int K(x, t)f(J_x(t))dt, \qquad \int K(x, t)f(t^3)dt. \tag{4c}$$

It is possible to deal with such transforms as follows. In each case of (3) a two-dimensional variable-density photographic plate is first prepared for the function $k(\zeta, \eta)$, in the way described in Sec. 4*a*, and



(*a*) photographic plate of opacity $k(\zeta,\eta)$ at the point (ζ,η)

(*b*) one-variable function-generator (see Sec. 4*b*) for the function $f(t)$ to be transformed

(*c*) the electron-beam scanning the plate of (*a*) moves along the locus $\eta = f(\zeta)$, thereby generating $k(t, f(t))$

FIG. 3. Method for obtaining non-linear transforms of the Hammerstein type (3*a*): $\int K(x, t)k(t, f(t))dt$.

a one-variable function-generator is set up for $f(\tau)$. For case (3*a*), τ is set equal to t. The plate for $k(\zeta, \eta)$ is scanned, in 1/1000 second, along the locus $\eta = f(t)$ (see Fig. 3); this is done by applying a saw tooth voltage $\zeta = t$ to the horizontal plates of the scanner for $k(\zeta, \eta)$ and a voltage proportional to $f(t)$, obtained from the one-variable function-generator for $f(t)$, to the vertical plates. The function $k(t, f(t))$ so obtained is then combined with $K(x, t)$, as in Sec. 3, to yield (3*a*).

For case (3*b*), τ is set equal to t, ζ to x, and η to $f(t)$. This last is done by scanning $k(x, \eta)$ along vertical strips, while using for the vertical deflection not a sawtooth voltage but instead the voltage $\eta = f(t)$ determined from the one-variable function-generator for $f(t)$. The result is $k(x, f(t))$; this is combined with $K(x, t)$ to yield (3*b*).

[3] Transforms of type (3*a*) are said to be of the "Hammerstein type": Hammerstein, "Nichtlineare Integralgleichungen nebst Anwendungen," *Acta Mathematica*, Vol. 54, pp. 117–176 (1930). See in particular pp. 123, 130. See also Nazarov, "Non-linear integral equations of Hammerstein's type," *Acta* (*Trudi*) *Universitatis Asiae Mediae*, Ser. V-a, Fasc. 33 (1941), 79 pp., which includes an extensive bibliography on non-linear integral equations.

For case (3c), ξ is set equal to x and η to t, but τ is set equal to $k(x, t)$; this means that the horizontal axis of the one-variable function-generator for $f(\tau)$ is not deflected at uniform speed by a sawtooth voltage, but instead by the voltage $\tau = k(x, t)$, thereby yielding $f(k(x, t))$.

II. INTEGRAL TRANSFORMS

7. THE FOURIER TRANSFORMS

The oldest and most common integral transform is the Fourier transform. The Fourier cosine transform is the special case of (1) for which we may take $(a, b) = (-\pi, +\pi)$, and

$$K(x, t) = \cos xt, \qquad x = 0, 1/10, 2/10, \cdots, 99/10. \tag{5}$$

A photograph of the kernel consists of a collection of vertical strips having cosinusoidal density distributions of frequency x, as shown in Fig. 4.

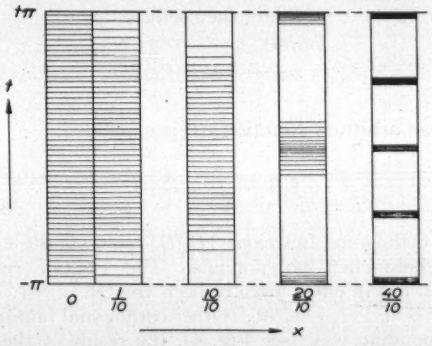


FIG. 4. Photograph of kernel $K(x,t) = \cos xt$ for Fourier cosine transform.

With the cosine transform photographic plate it will thus be possible to have the cosine transform of an arbitrary real function displayed on a cathode-ray tube screen essentially as quickly as one can draw a graph of the function.

7.1. Impulse Response of a Network

In particular the impulse response of an electrical network could be presented very quickly on a cathode-ray tube screen, as a consequence of the fact that the impulse response of a network is proportional to the cosine transform of the real part of the complex system function.

7.2. Determination of the Imaginary Part of a Network from the Real Part

Sine transforms can clearly be obtained in a way similar to that employed for cosine transforms. By combining sine and cosine trans-

forms we obtain the possibility of very rapid determination of the imaginary part of an impedance function from its real part (see also Sec. 9), since the imaginary part is the negative sine-transform of the cosine-transform of the real part:

$$I(\omega) = -\frac{2}{\pi}\int_0^\infty \left\{ \int_0^\infty R(\xi) \cos \xi t \, d\xi \right\} \sin \omega t \, dt. \tag{6}$$

7.3. Determination of the Phase of a Network from its Attenuation

Similar means can be used to determine the phase of a network from its attenuation, for minimum phase-shift networks, inasmuch as the attenuation and phase are the real and imaginary parts of the logarithm of the complex system-function.[4]

8. EXPANSION OF A FUNCTION IN ORTHOGONAL SERIES

One often wants to calculate the coefficients

$$a_n = \int P_n(t) f(t) dt \tag{7}$$

arising when an arbitrary function $f(t)$ is expanded,

$$f(t) \sim \sum_{n=1}^{\infty} a_n P_n(t), \tag{8}$$

in a series of orthogonal functions $\{P_n(t)\}$, such as the Laguerre, Hermite, or Tschebyscheff polynomials. The Fourier case, in which $P_n(t) = \cos nt$, has in effect already been treated in Sec. 7, if x is there replaced by $n = 0, 1, 2, \cdots, 99$. Other orthogonal families are treated in the corresponding way (see Fig. 4), the result of the computation being a cathode-ray graph of a_n against n.

For such problems a library of photographic kernels labelled "Laguerre Polynomials," "Jacobi Polynomials," etc., would be very useful: with the proposed instrument and such a library it would be the work of a few minutes to obtain the Laguerre expansions of several different functions, and then, if desired, to obtain their Tschebyscheff expansions, say, upon changing a photographic plate.

9. THE HILBERT TRANSFORM

Let $f(z) = u(\sigma, \omega) + iv(\sigma, \omega)$, where $z = \sigma + i\omega$, be a function analytic in a half plane, say the right half plane $\sigma > 0$. Then as has long been known, $u(0, \omega)$ and $v(0, \omega)$, which are the real and imaginary parts of $f(z)$ evaluated along the boundary of the half plane of analyticity,

[4] Y. W. LEE, *Jour. Mathematics and Physics* (Massachusetts Institute of Technology), June, 1932.

determine each other.[5] The above relation between $u(0, \omega)$ and $v(0, \omega)$ is precisely that existing between the real and imaginary parts of an electrical network function, and has been put to important use by Bode[6] in his work on the stability of feedback amplifiers.

One relation between the real and imaginary parts of a network function has already been given, in Eq. 6. Other relations are also well adapted to calculation by means of the integral transform computer. Thus consider the Hilbert transform:[7]

$$I(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{R(t)}{t - \omega}\, dt \tag{9}$$

between, for example, the real part $R(\omega)$ and the imaginary part $I(\omega)$ of an electrical network (the integral (9) is taken as a Cauchy principal-value at $t = \omega$). This is a special case of the integral transform formula (1), in which the kernel $K(\omega, t)$ is the easily approximated function[8] $1/(t - \omega)$.

10. CONVOLUTION INTEGRAL

The *convolution* of two functions $f(t)$ and $g(t)$ is the function

$$F(x) = \int_{-\infty}^{\infty} f(t)g(x - t)dt; \tag{10}$$

(10) is useful in electrotechnique, for example, because it is the impulse response of the cascade of two networks whose individual impulse responses are $f(t)$ and $g(t)$, or equivalently the response to a driving force $f(t)$ of a system whose impulse response is $g(t)$.

Equation 10 is the special case of Eq. 1 for which $K(x, t) = g(x - t)$. It would be possible to prepare a two-variable function table for the kernel $g(x - t)$ from a knowledge of $g(t)$, and then to proceed as outlined in Sec. 3. A serious disadvantage of such a procedure is that a two-variable function table $g(x - t)$ would then have to be prepared for each function $g(t)$. This can be avoided, however, by constructing an auxiliary and very simple Kronecker-δ kernel, namely

$$\delta(x, t) = \begin{matrix} 1 \text{ for } x = t, \\ 0 \text{ for } x \neq t, \end{matrix} \tag{11}$$

whose photograph is shown in Fig. 5, and then using $\delta(x, t)$ as a trigger

[5] E. C. TITCHMARSCH, "Introduction to the Theory of Fourier Integrals," Oxford, 1937; in particular Chapter V: Hilbert Transforms.

[6] H. W. BODE, "Network Analysis and Feedback Amplifier Design," van Nostrand, 1945; in particular Chapter XIV: Relations between Real and Imaginary Components of Network Functions.

[7] TITCHMARSH, *loc. cit.*, p. 120, formula 5.1.11.

[8] It is moreover possible to obtain the kernel $1/(t-\omega)$ by combining the Kronecker-δ kernel, explained in Sec. 10, with a one-variable function-generator for the function $1/t$.

to initiate the scanning of the one-variable function-generator $g(-t)$. Once the photograph of $\delta(x, t)$ is constructed, a convolution integral will require only one-variable function-generators.

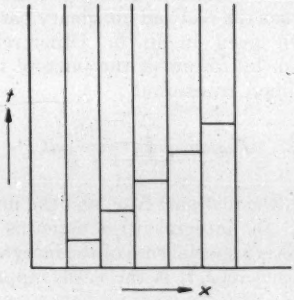


FIG. 5. Photograph of the auxiliary Kronecker-δ kernel used for convolution integrals.

III. INTEGRAL EQUATIONS

11. GENERAL METHOD

Solution of an integral equation usually involves a process of successive approximations. The proposed device for electronically computing the integral transform of a function materially expedites such procedures, because it makes possible direct and almost instantaneous comparisons between (*a*) a graph of the given function $g(x)$ and the transform of a trial unknown function $\varphi(t)$, as in Fredholm's linear integral equation of the first kind,

$$g(x) = \int_a^b K(x, t)\varphi(t)dt, \tag{12}$$

or (*b*) the graph of a trial solution $\varphi(t)$ and its integral transform, as in the homogeneous equation

$$\varphi(x) = \lambda \int_a^b K(x, t)\varphi(t)dt. \tag{13}$$

When the agreement between the two sides of Eq. 12 or 13 is sufficiently close for the problem at hand, the process of successive approximation is stopped.

It is easy to superimpose optically on a single screen the right and left hand sides of (12) (or (13)). Thus at each stage of the approximation process there is available an immediate and positive check which shows how close one has come to the solution, and eliminates all possibility of undetected mathematical errors.

If no *a priori* approximation to the solution is known, one can always

be found (in the computationally important case in which a unique solution exists), by replacing the integral by an approximating sum, following Volterra and Fredholm. Thus for (12) we have

$$g(x) \cong \sum_{i=1}^{n} \frac{b-a}{n} K(x, t_i)\varphi(t_i), \tag{14}$$

where the t_i are n intermediate points between a and b. Taking n intermediate values for x yields the family of simultaneous linear algebraic equations (see Sec. 12)

$$g(x_j) \cong \sum_{i=1}^{n} \frac{b-a}{n} K(x_j, t_i)\varphi(t_i) \qquad (j = 1, \cdots, n), \tag{15}$$

from which the approximate values of $\varphi(t_i)$ can be determined. From the values of $\varphi(t_i)$ a rough interpolation yields an approximation to $\varphi(t)$ that suffices to start a sequence of successive approximations using the integral-transform computer and based on the exact Eq. 12.

12. SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS

In many cases the solution of an integral equation can be systematized in a way ideally suited to the potentialities of the proposed instrument. As a first example consider n simultaneous linear algebraic equations in n unknowns,

$$\sum_{j=1}^{n} a_{ij}x_j = b_i, \qquad i = 1, \cdots, n. \tag{16}$$

((16) is, of course the special case of (1), with discontinuous kernel, that forms the basis of Fredholm's theory of integral equations.) With the instrument described in this paper, (16) can be solved as follows. First prepare a photographic plate for the matrix a_{ij}, as in Sec. 4*a*. For any given trial-vector $x_1', x_2', \cdots x_n'$, the error-vector

$$\epsilon_i = \sum_{j=1}^{n} a_{ij}x_j' - b_i, \qquad i = 1, \cdots, n$$

is very quickly obtained (in 1/10 second or less) by means of the proposed computer. Then, following Murray's method of gradients,[9] we take any trial-vector $x_1^{(0)}, x_2^{(0)}, \cdots, x_n^{(0)}$, and by varying $x_1^{(0)}$ while holding $x_2^{(0)}, \cdots, x_n^{(0)}$ fixed, we minimize $\sum_i \epsilon_i^2$. If we denote the value of $x_1^{(0)}$ so obtained by $x_1^{(1)}$, we next minimize $\sum_i \epsilon_i^2$ by varying $x_2^{(0)}$ while holding $x_1^{(1)}, x_3^{(0)}, \cdots, x_n^{(0)}$ fixed, thus obtaining $x_2^{(1)}$. The coordinates of the solution-vector are successively approximated in this

[9] F. J. MURRAY, "The Theory of Mathematical Machines," New York, King's Crown Press, 1948, revised edition, p. III–18.

manner. The process converges to the correct value, as shown by Murray, for any non-singular matrix a_{ij}, whether positive-definite or not.

13. LIOUVILLE-NEUMANN SERIES

For a Fredholm integral equation of the second kind:

$$\varphi(x) = g(x) + \lambda \int_a^b K(x, t)\varphi(t)dt, \tag{17}$$

the Liouville-Neumann method of successive substitutions gives the solution $\varphi(x)$ as a power series in λ,

$$\varphi(x) = \varphi_0(x) + \lambda\varphi_1(x) + \lambda^2\varphi_2(x) + \cdots, \tag{18}$$

where the $\varphi_n(x)$ are the successive iterated transforms

$$\begin{aligned} \varphi_0(x) &= g(x), \\ \varphi_1(x) &= \int_a^b K(x, t)\varphi_0(t)dt, \\ \varphi_2(x) &= \int_a^b K(x, t)\varphi_1(t)dt, \\ &\cdots \end{aligned} \tag{19}$$

Equivalent to the series (18) is its sequence $\{\varphi^{(n)}(x)\}$ of partial sums, converging to $\varphi(x)$,

$$\varphi^{(n)}(x) \to \varphi(x). \tag{20}$$

This sequence can be obtained by taking $g(x)$ as the first approximation to the solution,

$$\varphi^{(0)}(x) = g(x), \tag{21}$$

and successively substituting in (17):

$$\begin{aligned} \varphi^{(1)}(x) &= g(x) + \lambda \int_a^b K(x, t)\varphi^{(0)}(t)dt, \\ \varphi^{(2)}(x) &= g(x) + \lambda \int_a^b K(x, t)\varphi^{(1)}(t)dt, \\ &\cdots \end{aligned} \tag{22}$$

We have already seen, in Sec. 5, how well the proposed integral-transform computer is adapted to the iteration of transforms[10] needed for either (19) or (22).

[10] For practically calculating the series (18) or the sequence (20) there is required also an adder, that is, a device for yielding the sum of two voltages. Such a unit, like the integrator (item (d) in Sec. 4), happens to be very simple electrically. Also simple is a differentiator, if one should be needed.

It is now necessary to discuss the convergence of (18) and (20). As is well known, (18) represents an entire function of λ (in the complex-variable sense), and hence converges for all λ, in the important Volterra case for which

$$K(x, t) = 0 \text{ for } x < t; \tag{23}$$

hence the Volterra equation with variable upper limit,

$$\varphi(x) = g(x) + \lambda \int_0^x K(x, t)\varphi(t)dt, \tag{24}$$

is always solvable with the proposed computer by means of the Liouville-Neumann method.

For kernels $K(x, t)$ not satisfying (23), however, the series (18) generates a function that is not entire but only meromorphic, with poles at the characteristic values of λ. The radius of convergence is $|\lambda_1|$, where λ_1 is the characteristic value of least modulus. The solution of (17) by the Liouville-Neumann procedure, in the exact form given by (18) and (19), or (20) and (21), is hence limited to the range $|\lambda| < |\lambda_1|$. A suitable modification of the Liouville-Neumann method, well adapted to the integral-transform computer, is, however, competent to solve (17) for $|\lambda| > |\lambda_1|$; it is discussed in Sec. 14.

14. GENERAL SOLUTION OF THE FREDHOLM INTEGRAL EQUATION OF THE SECOND KIND

It was Fredholm's achievement[11] to give a general solution of (17) valid for all λ. Computationally, however, Fredholm's solution is inconvenient. One can attain the same result by modifying the method of successive substitutions, (21) and (22), in only one respect: instead of starting with $g(x)$ as the first approximation to $\varphi(x)$ (which is good enough when $|\lambda| < |\lambda_1|$), we must find a closer approximation $\varphi^{(0)}(x)$ as starting point. But this can always be done, by replacing the integral in (17) by an approximating sum (compare (14) and (15)) and thereby approximating to (17), in Fredholm's familiar manner, by linear equations:

$$\varphi(x_j) \cong g(x_j) + \lambda \sum_{i=1}^{n} \frac{b - a}{n} K(x_j, t_i)\varphi(t_i), \qquad (j = 1, \cdots, n). \tag{25}$$

(To solve (25) we must set $x_j = t_j$, $j = 1, \cdots, n$; this was not necessary in (15).) Equations (25) are solved by using the computer in the way described in Sec. 12.

Once the values $\varphi(t_i)$ are calculated we can avail ourselves of an idea due to Nyström[12] which gives us $\varphi(x)$ without interpolation (compare

[11] "Sur une classe d'équations fonctionelles," *Acta Mathematica*, Vol. 27, pp. 365–390 (1903).

[12] E. J. NYSTRÖM, "Über die praktische Auflösung von Integralgleichungen," *Acta Mathematica*, Vol. 54, pp. 184–204 (1930). Nyström points out the advantages in accuracy, for a given n in (25), of employing the Gauss or Tschebyscheff numerical integration formulas to select

the sentence following (15)):

$$\varphi(x) \cong g(x) + \lambda \sum_{i=1}^{n} \frac{b-a}{n} K(x, t_i)\varphi(t_i). \tag{26}$$

It is now this $\varphi(x)$ that we take as the $\varphi^{(0)}(x)$ to start a Liouville-Neumann process (22).

From Fredholm's existence theorems we know that the sequence (22) so obtained will converge if n in (25) is large enough. Just how large n must be, it is, of course, not possible to say in general, but from Nyström's paper one can conjecture that for many practical cases $n = 6$ or 8 should suffice. In any case our instrument provides us with an instantaneous check, as already mentioned in Sec. 11, from which we can always determine whether the approximation is adequate.

15. A BOUNDARY-VALUE PROBLEM LEADING TO A NON-LINEAR INTEGRAL EQUATION

Duffing's non-linear ordinary differential equation problem for the forced oscillations of a pendulum,

$$\frac{d^2\varphi}{dt^2} + \alpha^2 \sin \varphi(t) = f(t), \qquad \varphi(0) = \varphi(\pi) = 0, \tag{27}$$

with $f(t)$ a periodic forcing function of period 2π, has been converted by Hamel[13] to the non-linear integral equation of the second kind,

$$\varphi(t) = g(t) + \alpha^2 \int_0^{\pi} K(t, \tau) \sin \varphi(\tau) d\tau, \tag{28}$$

with symmetric kernel[14]

$$K(t, \tau) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{\sin nt \sin n\tau}{n^2} = \begin{cases} t\left(1 - \dfrac{\tau}{\pi}\right), & t < \tau, \\ \tau\left(1 - \dfrac{t}{\pi}\right), & t > \tau, \end{cases} \tag{29}$$

and known

$$g(t) = -\int_0^{\pi} K(t, \tau) f(\tau) d\tau. \tag{30}$$

The transform in (28) is of the non-linear type (4a), already discussed in Sec. 6. The solution of (28) for $\alpha^2 < 1$ has been shown by

the intermediate points t_i and assign them suitable weights. This advantage was especially important to Nyström because (a) the $\varphi(x)$ determined from (26) is his final answer, and (b) the computations are carried out by ordinary means. But in our case, where the result of (26) is to be refined by further steps, it seems simpler to take the points t_i equally spaced and equally weighted.

[13] "Integralgleichungen," Berlin, Springer, 1937, in particular pp. 158–161.

[14] The photographic plate for this kernel, which is quite common in the applications, is especially simple to construct.

Hamel to be given by a Liouville-Neumann method of successive approximations, $\varphi^{(n)}(t) \to \varphi(t)$, with

$$\varphi^{(0)}(t) = g(t),$$

. . . . . .

$$\varphi^{(n)}(t) = g(t) + \alpha^2 \int_0^\pi K(t, \tau) \sin \varphi^{(n-1)}(\tau) d\tau. \tag{31}$$

The successive approximations are well adapted to the proposed instrument, which is not inconvenienced by the non-linear character of the transforms involved in (31).

16. THE DIRICHLET BOUNDARY-VALUE PROBLEM OF PLANE POTENTIAL THEORY

Perhaps the most common application of integral equations is to the Green function method of solving boundary-value problems. A classical illustration of this method arises in the Dirichlet problem in the plane. Here we are given a closed contour C in parametric form,

$$x = x(s), \qquad y = y(s), \qquad a \leqq s \leqq b, \tag{32}$$

(we limit ourselves to the case in which C is convex and has only a finite number of corners), and a prescribed function $g(s)$, and have to determine a function $u(x, y)$ that is harmonic in the interior of C,

$$\Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \tag{33}$$

and takes on the values

$$u(x(s), y(s)) = g(s) \tag{34}$$

on the boundary C.

Let $P(x, y)$ be an interior point and

$$\theta_P(s) = \tan^{-1} \frac{y(s) - y}{x(s) - x}, \tag{35}$$

the angle between the x-axis and the vector joining P to the running point $(x(s), y(s))$ on C. Then the logarithmic potential induced by a dipole layer of moment $\mu(s)$ is[15]

$$u(P) = u(x, y) = \int_a^b \mu(s) \frac{\partial \theta_P(s)}{\partial s} ds. \tag{36}$$

The Dirichlet problem can then be reduced to determining a function $\mu(s)$ so that for the function $u(x, y)$ defined by (36) we have that $\lim_{P \to C} u$ satisfies (34).

In one of the earliest investigations in integral equation theory,

[15] If we write $\theta = \theta_P$ and denote by $s = s_P(\theta)$ the functional relationship between s and θ_P, we can also write (36) in the form

$$u(P) = \int_0^{2\pi} \bar{\mu}(\theta) d\theta$$

where $\bar{\mu}(\theta) = \mu(s_P(\theta))$.

Carl Neumann showed[16] that $\mu(s)$ can be determined by solving the integral equation

$$2\pi\mu(s) + \int_a^b [\mu(t) - \mu(s)] \frac{\partial \theta_s(t)}{\partial t}\, dt = g(s), \tag{37}$$

where

$$\theta_s(t) = \tan^{-1} \frac{y(t) - y(s)}{x(t) - x(s)} \tag{38}$$

is the angle between the x-axis and the vector joining the point $(x(s), y(s))$ on C to the running point $(x(t), y(t))$ on C. He showed further that (37) has the solution (original Neumann series)

$$\mu(s) = \frac{1}{2\pi} [\mu_0(s) + \mu_1(s) + \cdots] \tag{39}$$

with

$$\mu_0(s) = g(s) \tag{40}$$

and

$$\mu_n(s) = \frac{1}{2\pi} \int_a^b [\mu_{n-1}(t) - \mu_{n-1}(s)] \frac{\partial \theta_s(t)}{\partial t}\, dt. \tag{41}$$

Neumann showed, finally, that (39) converges for all convex contours with a finite number of corners.[17]

Hence (39) provides a systematic solution procedure of general validity. It is exceptionally well adapted to the capabilities of the proposed integral-transform computer, as will now be shown.

Instead of constructing a separate kernel

$$\frac{\partial \theta_s(t)}{\partial t} \tag{42}$$

for each contour (32), which might require some hours per contour, we instead construct once and for all, for the whole class of Dirichlet problems, the simple auxiliary kernel

$$k(\zeta, \eta) = \text{angle } (\zeta, \eta) = \tan^{-1} \frac{\eta}{\zeta}. \tag{43}$$

Having done this, we now employ the one-variable function generators for $x = x(t)$ and $y = y(t)$ (which, we recall, require only a few seconds to alter) in order to set $\zeta_s(t) = x(t) - x(s)$ and $\eta_s(t) = y(t) - y(s)$ into (43). Combining this with a t-differentiator[18] yields the kernel (42).

[16] "Untersuchungen über das logarithmische und Newtonsche Potential," Leipzig, 1877.

[17] If there are no corners the equations are simplified, for then

$$\int_a^b \frac{\partial \theta_s(t)}{\partial t}\, dt = \pi$$

and (37), for example, becomes

$$\pi\mu(s) + \int_a^b \mu(t) \frac{\partial \theta_s(t)}{\partial t}\, dt = g(s).$$

[18] See footnote 10.

With this method only a few seconds are therefore required to change the Dirichlet-problem integral-equation from one contour to another.

The dipole moment $\mu(s)$ is now determined by using the integral-transform computer to solve (37), according to (39) and the recursion formulas (40) and (41).

For many applications the determination of the dipole moment $\mu(s)$ can be regarded as a complete solution of the Dirichlet problem. Sometimes, however, it is desirable to display the form of the potential function $u(x, y)$ itself. This is easily done, as follows. By employing the angle kernel (43), with $\zeta_P(s) = x(s) - x$ and $\eta_P(s) = y(s) - y$, we obtain $\theta_P(s)$ (see (35)). A differentiation and multiplication is followed by a definite integration to yield $u(x, y)$ according to (36). The process of determining $u(x, y)$ takes 1/1000 second per point (x, y).

If desired the interior of the contour C can indeed be scanned along successive parallel lines to give a 'television picture' of the potential function $u(x, y)$, the picture being bright where $u(x, y)$ is large, dim where $u(x, y)$ is small. For 100×100 points in the picture of $u(x, y)$ a time of 10 seconds should suffice, once the dipole moment $\mu(s)$ is known.

Illumination Meter Records Changing Light Values.—Rapid fluctuations in illumination intensity are accurately recorded by a new photoelectric light recorder introduced by the Weston Electrical Instrument Corp. Developed originally to chart the fast-changing light output of railway fuses, this new instrument also lends itself to many applications where wide variations in light intensity must be recorded faster than can be done by hand.

Though a new device, the Weston photoelectric light recorder utilizes principles previously employed in other Weston and TAGliabue instruments. The sensing element, which can be placed wherever convenient, employs a "Photronic" photoelectric cell of the dry disk type made by Weston for more than 15 years. A built-in "Viscor" filter matches its color response to that of the human eye. Its "Celectray" recorder, of proved TAGliabue design, follows rapid changes without overshooting or hunting.

Full scale values of 20, 40 or 80 foot-candles are selected by a single switch. The recorder can also measure candlepower of any value by proper adjustment of distance from the light source. Descriptive literature giving technical details and specifications is available from the manufacturer.

GFB-NBS Strain Tester for Rubber and Rubber-Like Compounds.—The new GFB–NBS strain tester is an automatic precision testing machine for the evaluation of rubber and rubber-like compounds. Developed and constructed in 1948 by G. F. Bush Associates, in close cooperation with the National Bureau of Standards, it has satisfied the most exacting requirements in production and laboratory control and testing programs. For example, its high precision will easily detect variations within a single 6-in. square sheet of vulcanized rubber.

This tester measures the percentage elongation of a virtually motionless specimen at a definite time after the application of a pre-chosen stress. This stress is obtained by loading the specimen friction-free in proportion to its thickness in millimeters. It predicts cure time to give any desired percentage elongation. Precise measurements of extension set for a specified stress may be obtained since specimens can be stressed for a uniform time interval. Effect of storage on elasticity can now be obtained from one batch per cure.

The machine is automatic, and any unskilled operator can easily obtain all data needed for evaluations. The automatic property of this tester reduces human errors to a minimum, and a timer accurately controls all components of an operation cycle. The maximum operation period is only 80 seconds. The protective controls give warning when an operation is abnormal, indicating that the data for that particular cycle are not to be used. Also, no change in weight loadings for a particular cycle is possible since a solenoid locks the weight combination.

A retractable screen grid for jet air intakes, which in one test stopped a .50-cal. shell case from entering the engine at an air speed of 625 mph., has been designed for the Air Materiel Command, Dayton, Ohio, by the University of Kentucky's Aeronautical Research Laboratory. AMC engineers estimate that a small object scooped into a jet intake at speeds over 600 mph. has a velocity about equal to that of a .45-cal. bullet fired into a stationary object. Thrust loss due to the grid is only 6 per cent.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

HIGH-SPEED REVERSAL OF ELECTRIC MOTORS

A method for reversing a small electric motor in three to four milliseconds has been developed by Jacob Rabinow at the National Bureau of Standards. Designed specifically to meet the need for high-speed reversal of magnetic tapes in the memories of electronic digital computing machines, the technique may prove useful in many other applications.

While it is relatively easy to stop a motor quickly by the use of brakes, rapid starts in either the same or the reverse direction are limited by the low torques which can be obtained by electromagnetic means alone. In the Bureau's rapid-reversal motor, the kinetic energy of the rotor, instead of being dissipated as heat in a brake during deceleration, is converted into potential energy in a spring, which is then used to accelerate the rotor rapidly in the opposite direction.

A small low-inertia two-phase motor operating at 3200 rpm. was used. The reversal spring consists of a steel torsion bar approximately 31 inches long and $\frac{3}{16}$-in. in diameter. Only one phase of the motor is connected to the a-c. power supply; thus the motor will rotate in the starting direction, either clockwise or counterclockwise. The motor shaft is rigidly connected to one end of the torsion bar, which is equipped at the other end with two positive unidirectional clutches. One clutch prevents clockwise, and the other counterclockwise, rotation. If the motor is rotating in a clockwise direction and the proper clutch is engaged, the adjacent end of the torsion bar is thereby stopped; this brings the rotor to a stop in approximately 20 degrees. The potential energy stored in the torsion bar is then returned to the rotor in the form of a counterclockwise impulse. The motor attains virtually full speed in the new direction within about two millisceonds.

Two methods were used in the Bureau's investigation to check the reversing time of the motor. For the simpler test, a black disk provided with a white index-line was mounted on one end of the motor shaft and photographed against a circular scale, using a high-speed motion-picture camera at approximately 4000 frames per second. When the film is run through a projector at the normal rate of 16 frames per second, the behavior of the disk is easily observed. When the motor is rapidly reversed, the backward speed is, for a short time, slightly lower than the speed before reversal because the ball-bearing assembly does not reverse its rotation as rapidly as does the motor shaft.

A second means of measuring reversal speed, suggested by Frank

* Communicated by the Director.

Shepard of the Shepard Laboratories, involves locking the far end of the shaft and applying direct current to one winding of the motor, while alternating current is supplied to the other winding. Oscillations are thus induced in the rotor-torsion bar. Varying the frequency of the current makes it possible to determine the frequency of torsional resonance. The time required for the rotor to reverse can be easily computed since it is one-half of one period of a torsional oscillation at resonance. The resonance in the system tested occurred at approximately 125 cps., indicating (in a period of 8 milliseconds) a reversal time of 4 milliseconds.

In the experimental model of the rapid-reversal mechanism built at the Bureau, the clutches are operated manually, but it is expected that in normal use the clutch mechanism will be operated by suitable electromagnetic controls. Although the studies thus far have used a small motor of about $\frac{1}{75}$ hp, it is anticipated that motors of all sizes could be reversed rapidly by this technique, the speed of reversal being limited only by the mechanical strength of the various parts. Moreover, by use of suitable circuitry, rapid reversal of motors can be obtained without producing large current surges in the electrical supply lines.

It would be entirely feasible to utilize other types of energy-storing devices, such as coil springs or pneumatic cylinders, instead of the torsion shaft. It would also be possible to use a stationary energy-storing system and to locate the reversing clutches between it and the motor shaft.

EFFECT OF FUEL IMMERSION ON LAMINATED PLASTICS

Changes in weight and in the dimensional and flexural properties of laminated plastics immersed in three hydrocarbon liquids have been measured at the National Bureau of Standards as a means of evaluating the relative resistance of the laminates to different fuels. Constituting one phase of a continuing program on the properties of laminates for a wide variety of aircraft applications, the tests were designed to provide data needed in the preparation of specifications for those plastics found suitable for use as auxiliary (droppable) fuel tanks, radomes, instrument panels, and containers. The study,[1] conducted under the sponsorship of the National Advisory Committee for Aeronautics by W. A. Crouse, Margie Carickhoff, and Margaret A. Fisher of the Bureau's organic plastics laboratory, involved the determination of the effects on representative laminated plastics (see Table I) of cyclic and continuous immersion in three fuels.

The cyclic immersion test consisted of 10 alternate 24-hr. periods of fuel-immersion and air-drying. Weight, dimensional measurements, and flexural determinations were made after reconditioning at 77°F. and

[1] For further technical details see forthcoming NACA Technical Note, "Effect of fuel-immersion on laminated plastics," by William A. Crouse, Margie Carickhoff, and Margaret A. Fisher.

50 per cent relative humidity for 48 hr. For the continuous immersion test, duplicate sets of specimens were immersed for 7, 30, 90, 180, and 360 days. On one set of specimens weight, dimensional, and flexural measurements were made immediately after removal of the specimens from the fuel and on a second set, after reconditioning for 7 days at 77°F. and 50 per cent relative humidity. The fuels used for these tests were heptane (an aliphatic hydrocarbon), toluene (an aromatic hydrocarbon), and SR-6, a representative aircraft test fuel composed of blended aliphatic and aromatic hydrocarbons.

Dimensional and flexural strength changes are the principal properties to be considered in many of the aircraft applications. Most of the materials may be used for this purpose. Considering only the 360-day continuous immersion test, a cotton-fabric phenolic laminate exhibited the least changes in dimensions, and a paper phenolic laminate showed the smallest changes in flexural strength in the three fuels.

In all three fuels the weight changes of the majority of the laminates were less than 1.0 per cent in the cyclic test, and did not exceed 1.5 per cent after 360 days of continuous immersion. The largest weight changes

TABLE I.—*Description of materials*

Type of Laminate	Average thickness (inch)	Average density (g/cm^3)	Number of plies
Lignin; paper	0.128	1.38	—
Melamine; cotton fabric	0.145	1.47	11
Phenolic; asbestos fabric	0.145	1.49	5
Phenolic; cotton fabric	0.124	1.35	7
Phenolic; rayon fabric	0.160	1.35	6
Phenolic; paper	0.122	1.42	54
Polyester; cotton fabric	0.147	1.36	6
Polyester; glass fabric	0.125	1.64	6

were usually obtained in toluene. With very few exceptions, the length and width changes after either the cyclic or the 360-day immersion in the fuels did not exceed 0.1 per cent. In both types of test the changes in thickness were not over one per cent.

The best weight and dimensional stability in all three fuels in the cyclic test was observed with the glass-fabric unsaturated-polyester laminate. After the 360-day immersion test, the weight and dimensional changes were, in the majority of cases, equal to or higher for the samples that were tested immediately than for those that were reconditioned before testing.

The changes in flexural strength for the cyclic test and after the 360-day immersion test were, in most cases, negative. However, the losses were not over 10 per cent for all of the samples.

The changes in flexural modulus of elasticity were, in the majority of cases, negative in the cyclic and the continuous immersion tests. In the

cyclic test losses greater than 10 per cent were shown only by the cotton-fabric unsaturated-polyester sample. After 360 days of immersion no losses greater than 10 per cent occurred.

Since there may be appreciable differences in the properties of individual sheets taken from the same batch, in different batches made by the same manufacturer, and even in similar laminates made by different manufacturers, only general inferences about the different types of laminate can be drawn from these data. No single sample exhibited minimum changes in all of the properties, under the different conditions of immersion and with the three different fuels.

THE FRANKLIN INSTITUTE

ANNUAL REPORT OF THE BOARD OF MANAGERS FOR THE YEAR 1949

The President, in behalf of the Board of Managers, presents that Board's Annual Report for the calendar year 1949 as follows:

Changes in Board of Managers

During 1949 the Institute had the misfortune to lose three members of its Board of Managers by death—Messrs. Malcolm Lloyd, Jr., Philip C. Staples, and James S. Rogers—and they and their valuable services will be greatly missed. Messrs. Francis J. Chesterman, E. Paul du Pont, and S. Wyman Rolph were elected to fill the vacancies. Messrs. William J. Meinel and Edward G. Budd, Jr. were elected at the annual meeting of the Institute to fill vacancies on the Board then existing.

Mr. Walton Forstall resigned as a Vice-President and was elected a Manager to fill the vacancy occurring when Dr. G. H. Clamer, a Manager, was elected a Vice-President.

Mr. Orus J. Matthews resigned from the Board due to press of other responsibilities and Mr. John Russell, Jr., was elected to fill this vacancy.

Mr. Lawrence LePage came on the Board as an *ex-officio* member upon his election as Chairman of the Committee on Science and the Arts.

Membership

The total membership at the end of 1949 was 5941. During the 125th Anniversary Year many members of the Institute continued their excellent cooperation by suggesting names of persons they felt would be interested in membership. This friendly assistance of our members helped make it possible to elect 932 new members, with a net gain of 55 for the year 1949.

The "Science is Fun" program of popular talks, preceded by informal dinners, has proved to be of continued interest to the members and their friends.

Friends of Franklin

The group of over 200 individuals and companies comprising the Friends of Franklin has continued its generous support. The amount contributed by them in 1949 was $24,055.00.

Meetings

During the year seventeen meetings, either stated meetings of the Institute or joint meetings with technical societies, were held at which the average attendance was 222. The dinners preceding these meetings were well attended by an average of 96 members and their guests. The first lecture under the "Edward G. Budd Lecture Foundation" was given by Dr. George L. Kelley of Oxford, England, on the subject "Life and Work of Edward G. Budd."

The twelfth series of piano lecture-recitals by Mr. Guy Marriner continued into the spring of 1949; and the thirteenth series opened in October 1949 and has held the same appeal for music lovers as its predecessors. An increase in the admission charge was regretfully found necessary; and an experiment with season tickets was launched and found successful.

The Institute has continued to make its resources available for the meetings of other scientific and technical organizations, although a moderate increase in the nominal fee charged for its facilities became unavoidable in 1949 under the steady increase in operating costs that had occurred. Thirty-three organizations with aims and purposes paralleling those of our Institute came to it for a total of 143 meetings; these were in addition to the 95 meetings held by, or participated in by, the Institute.

Committee on Science and the Arts

The entire Science and Arts Committee met nine times, while twenty-nine Sub-Committee meetings were held. Thirteen Medals were awarded at the annual Medal Day Exercises, one of which was a Franklin Medal award to Professor The Svedberg. Three George Hoadley certificates were presented during the year to:

Mr. James S. Rogers, for forty-five years of service.
Dr. Winthrop R. Wright, as of January 1948, for twenty-five years of service.
Mr. Charles H. Masland, 2nd, for twenty-five years of service.

The Journal of The Franklin Institute

Our JOURNAL completed its 124th year of continuous publication in December 1949, maintaining its high standards of technical excellence in the twelve issues for the year. During 1949, 68 manuscripts were submitted for consideration, of which 10 were published and 38 were added to the lengthy list awaiting publication. The large backlog of papers made it advisable to add 138 pages of technical material to the 1949 JOURNAL (in comparison with 1948). In spite of this increase, the backlog has continued to accumulate until there are now enough manuscripts in the file to publish the JOURNAL for two years. This gives an indication of the esteem in which our JOURNAL is held, for all authors have been advised of this situation and have preferred delayed publication in the JOURNAL to earlier publication elsewhere.

During 1949 the following subdivisions were added to "The Franklin Institute" section, to place on permanent record the activities of the Institute departments mentioned: (1) The Franklin Institute Laboratories; (2) The Bartol Research Foundation; (3) The Membership Department; and (4) The Journal of The Franklin Institute.

The 4500 copies which are distributed monthly throughout 46 states, the District of Columbia, and 40 foreign countries, go to members, subscribers, Friends of Franklin, and to other publishers in exchange for their publications.

The April 1949 issue commemorated the 125th anniversary of the founding of the Institute by reproducing the "First Annual Report of the Proceedings of The Franklin Institute" from the original. About 2000 extra copies of this issue were distributed to men and women prominent in U. S. industrial, political and cultural life.

Library

During the year 1949 there have been added to the Library: 2428 volumes, 1326 pamphlets, and 654 maps. Many generous gifts were received from members of the Institute, from other scientific societies, and from friends of the Institute. 791 volumes were added through purchase and 709 volumes by gift. 1100 periodical publications were received regularly, of which 421 were in exchange for the JOURNAL. Exchange relations with many foreign societies that had terminated due to the war were reestablished. Complete patent specifications from the United States, Great Britain and Switzerland were received up to date. Many serial publications have been bound, and a number of volumes have been rebound.

Numerous requests by letters and 5547 telephone calls were received for information and research. 10,178 photostat prints of material in the Library collection were furnished to persons in both the United States and foreign countries. There were 6395 visits by members, 1424 visits by non-members, and 1753 visits by members of the staff of The Franklin Institute Laboratories, to make use of the Library collection.

The rare books of the Library have been placed in a separate bookcase in the Librarian's office. This collection constitutes a very fine representation of books in the field of early science and technology.

The Library endowment funds were enriched by a very generous addition to the Thorn Fund.

The total contents of the Library as of January 1st 1950 were: 133,529 volumes, 48,919 pamphlets, and 5321 maps.

Museum

Attendance:

The comparative attendances at the Museum and Planetarium were:

	Paid	*Free*	*Total*
For 1948	160,428	71,559	231,987
For 1949	167,844	76,588	244,432

As in former years all public and parochial school children from Philadelphia were admitted

to the Museum and Planetarium without charge when attending in groups accompanied by a teacher. When attending individually, the children paid only the Federal admission tax.

Exhibits:

A superb model railroad, lent by the Baltimore and Ohio Railroad, was temporarily displayed in the former Electrical Communications section. This model, representing the B. and O. Cumberland Division, was unusually complete and proved to be a source of delight to young and old.

In connection with the Institute's celebration of its 125th anniversary, opportunity was taken to commemorate the electrical discoveries of Benjamin Franklin, which began in 1746. An exhibition was arranged to demonstrate the fundamental laws of electricity and magnetism and to indicate the principal developments which have sprung from these laws. New techniques were tried in labelling this exhibit, which will remain on permanent display.

Science in lighter vein was presented in an attractive Illusions Show which, skillfully blending science and psychology, aroused favorable comment from psychologists all over the country.

The first annual Science Fair was held in Franklin Hall during the last week in May. Almost one hundred and fifty exhibits were submitted by science students in the Greater Philadelphia area, many of them being of superior merit. This response was sufficient to convince us and the cosponsor, *The Philadelphia Inquirer*, that the Science Fair will become a popular annual institution.

A special exhibition which aroused much interest among photographers was the display of the prize-winning entries for *Popular Photography*'s national contest. The local winners received their prizes on the opening day.

A separate section has been created called the Hall of Honor in which are assembled the original pieces of apparatus and associated articles owned by Benjamin Franklin, Priestley, Faraday, Edison, Elihu Thomson, etc. It is hoped to attract gifts of a similar authentic nature to increase this collection.

Visitors who find walking through the various sections of the Museum a fatiguing task will welcome the comfortable lounge which has been opened on the Ground Floor. Here, the footweary visitor may rest and smoke in pleasant surroundings, and enjoy television programs. The walls are decorated with photo-murals illustrating the work and equipment of the Institute's Laboratories for Research and Development. The lounge is conveniently situated to be used by audiences assembling for lectures. The television receiver has been lent by the R.C.A., Victor Division.

Additions:

Special care continues to be exercised in the selection of material offered for exhibition but during the year the Museum has received gifts or loans of sixty-two items. The most noteworthy of these was the second of the three wills known to have been written by Benjamin Franklin. This will, the gift of Mrs. Mary Curtis Zimbalist, is the subject of a brochure written by Carl Van Doren and published with a facsimile by the Institute.

Among other acquisitions are a phonograph and long playing records from Motor Parts Company; Victrola and records from the R. C. A., Victor Division; Link Trainer from the University of Pennsylvania; transparent tooth exhibit from the Philadelphia County Dental Society; shaded pole motors from Barber-Colman Company; personal weighing machine from Yale and Towne Manufacturing Company; Radiosonde from the U. S. Army, Evans Signal Laboratory; Model of British Merchant ship of 1690 from Professor C. M. Gay; and a scale model of a meteor crater from the D. M. Barringer estate.

Special Activities:

The radio program, Science is Fun, having completed its fifth series has been transformed into a television program. This program is prepared with the cooperation of station WFIL-TV and the public, parochial, and private schools of the area.

A new activity in which young people between the ages of eight and twelve were taken on a

series of especially conducted tours of the Museum and Planetarium was opened. Each tour embraced movies, demonstrations, games, and experiments built around a specific scientific topic. This activity was so warmly welcomed that a second series was organized, and others will follow.

A permanent change in the hours of the Planetarium demonstrations was inaugurated in the month of August. Although the night demonstrations were reduced to three a week, on Wednesdays, Fridays, and Saturdays, the attendance at individual demonstrations has shown an increase.

A new policy of Planetarium Lobby displays tied in with the current demonstrations has been started. Efforts to secure better publicity have been attempted through improved outdoor signs, in which three dimensional effects are used when possible, and through the willingness of the three Philadelphia metropolitan newspapers to provide more adequate coverage of the demonstrations. The results of these efforts are reflected in increased attendance at Planetarium lectures.

Personnel

The statistics for the year show 467 employees on January 1, 1949 and 407 on December 31st, as the result of 84 employments, 144 separations and 7 deaths.

The year saw the inauguration of a contributory life and sickness accident insurance plan.

Public Relations

During 1949 the press of economy dictated termination of the Institute's arrangements with the Public Relations consultants with whom it had had an association of long standing; and we have since been dependent upon our staff members, of whom only two could be assigned full time, and a very few could help part-time to report our activities to the public we exist to serve. The results of this cooperative effort have been favorable.

The Public Relations staff has carried on, extended, and added to, its usual activities; and has been called upon several hundred times to supply pertinent information in response to inquiries regarding the Institute or Benjamin Franklin.

The Institute News was published ten times during the year, two months having been covered by a single issue twice during the summer.

A new pamphlet entitled "This Is The Franklin Institute" was prepared to give the Institute's history, aims, and activities briefly and concisely; and was found so useful as quickly to exhaust the first printing of 10,000 copies and to require a reprinting within three months.

Hostess Committee

The Hostess Committee has continued to be of inestimable help in its many activities. It has been particularly effective in its effort to improve the appearance of the Institute grounds.

Research Laboratories

Franklin Institute Laboratories for Research and Development

The dollar volume of the research and development workload carried in 1949 increased slightly over the 1948 figure—to $1,743,714—while the number of persons employed by the Laboratories decreased from 246 to 220 during the year. There was a small increase in the space assigned to Laboratory activities to 57,000 sq. ft.

Success rewarded the continuing effort to place a progressively increasing portion of the Institute's research resources at the disposal of private industry, the percentage of total effort going to private instead of government projects having become 13.0 for 1949 as compared to 10.9 for 1948.

The number of proposals submitted in 1949, both for private and for government research work, was markedly greater than for 1948; and the percentage of acceptances was markedly smaller.

New industrial sponsors acquired during 1949 included Convertoplane Corporation, Hercules Powder Company, Electronic Tube Corporation, Renuzit Home Products Company, General Electric Company (Philadelphia), U. S. Pipe and Foundry Company, Raybestos-Manhattan, Inc., Armour Research Foundation, and Dodge Steel Company. Small initial projects

were also undertaken for Philadelphia Electric Company, Superior Tube Company, and Leeds and Northrup Company.

The three former technical divisions of the Laboratories were combined to form a single Research Operations Division under a Director of Research Operations, the former Division Directors becoming Associate Directors of the laboratories. The former New Projects Procurement Division became the Contracts Division, headed by the Assistant Director for Contract Administration; and the title of the Head of the Administration Division was changed to Manager of Administrative Services.

New Equipment

The more important items of new equipment acquired during the year include a high frequency oscillator in the range 20 to 100 kilocycles per second stable to one part in a million. This equipment is being used in our ultrasonic research, measuring the properties of liquids. A strobolume unit was obtained for use in the Photoelastic Laboratory, to produce a light source of very short duration at specified intervals for stroboscopic photography. A Western Electric Fastax high-speed motion picture camera, designed for a top speed of 4,000 frames per second, was also procured. A high-speed drum camera having greater flexibility in operation than any on the market was developed to study the mechanism of flame propagation. A test setup for Bentonite flow studies was made, and a 400-cycle controlled frequency generator was put into service.

Work Performed in 1949

(*a*) *Electrets:* Work has been progressing in the study of wax electrets investigating the role of the dielectric material in their production and behavior.

(*b*) *Bentonite Flow Studies:* To visualize disturbances in air or water by moving objects such as airplanes and ships, the Institute started an investigation of these flow problems by observing simulated models submerged in a moving stream of a colloidal suspension of Bentonite clay in water. The method can also be used to determine causes of excessive wear or erosion in flues or pipes, and to indicate improvements that will reduce such wear.

(*c*) *Projection Magnifier:* A projection magnifier was developed to the field testing stage for the W. K. Kellogg Foundation, Battle Creek, Michigan, as a device to aid the partially sighted. The Magnifier is a reading aid for persons requiring a greater magnification of reading matter than that offered by a simple magnifying glass, and is portable.

(*d*) *Sonic Tester:* A sonic tester was developed for the Simonds Abrasive Company of Philadelphia, Pennsylvania, to determine the quality of abrasive wheels by non-destructive tests. It also serves as an aid in simplifying the present system for grading abrasive wheels.

(*e*) *Other Completed Items* included Loran trainer transducers, a recording unit for underwater tests, four units of an airborne fire-control system, a parallax computer, and a radar modification.

(*f*) *Engineering Mechanics:* Work was conducted in experimental and analytical stress analysis, including photoelasticity, dynamic and static electric strain-gage techniques, and the use of brittle lacquers. Tests using these techniques were conducted to obtain strains and stresses caused in a torus of an omega-type expansion joint by both axial motion and internal pressure.

Other work in engineering mechanics dealt with applied hydraulics, including the study of packings, valves, and pump performance; hydraulic power transmission, its control and application to industrial machinery; theoretical and experimental studies in interior ballistics; and the development and application of oil film bearings.

(*g*) *Rocket Instrumentation:* This included developing, designing and manufacturing alterations to large missiles to provide for greater loads of upper atmosphere research instruments; equipment to be placed in rockets for securing upper atmosphere data; control mechanisms for both periodically and continuously operating instruments by either remote or local control; ejecting and separating mechanisms for removal of instrumentation from missiles for recovery by parachute; and containers for instrumentation and parachute deployment equipment. In

addition to the above, personnel have been trained to assist in the final preparation of rockets for firing at the proving grounds.

(*h*) *Solid State Physics:* The electronic properties of metals, semiconductors, and dielectrics, including development and test of special thermocouple materials for new applications; the mechanism of conduction and noise in resistors; and electrets and tests of dielectric materials were investigated.

Mechanical and material properties research included plastic deformation of metals, vibration problems, friction phenomena, development of improved industrial products of a colloidal nature (such as emulsions, ceramics, asphalts, adhesives, paints, etc.), and the application of rheological conceptions and methods to the development of improved plastic materials.

Research was also carried on in X-ray diffraction, electron microscopy, and metallography.

(*i*) *High Polymer Research:* Basic and applied research was conducted on polymer problems. The relationship between molecular structure and physical properties of polymeric materials was investigated as was the application of ultrasonic methods to the evaluation of polymer properties.

(*j*) *Radiation and Thermodynamics:* The activity in radiation and thermodynamics concerned itself largely with studies of combustion phenomena. In addition to all of the standard equipment employed, specialized spectrographic instrumentation was developed to investigate the kinetics of transient flames in the ultraviolet, visible, and infrared regions of the spectrum. This specialized equipment included a large aperture ultraviolet scanning spectrometer, a large aperture image orthicon spectrograph for the visible and infrared region, a ten-channel infrared spectrograph for studies up to 25 microns, and a highly versatile drum camera, made by the Laboratories as reported above. Studies were made on infrared detection, particularly with respect to the pneumatic detector; on optical and thermal properties of thin metallic films; on the thermal properties of solids, liquids, and gases; and on problems involving engineering thermodynamics and heat transfer.

(*k*) *Electronics:* The Laboratories worked extensively in electronics, having built analogue computing mechanisms, developed magnetic recording devices, designated and constructed simulators which combine analogue computing with portrayals of operating conditions and built the amplifying and driving equipment necessary for the computers, the recorders and the simulators. Certain testing and measuring devices were designed to employ sonic means for grading resinoid and ceramic products, and microwave and supersonic techniques were evolved for testing a range of metallic and non-metallic products. Further, pneumatic and mechanical computers and graphical simulators were built for studying problems of traffic flow.

(*l*) *Instruments and Controls:* Servo mechanisms of a variety of sizes were designed, built and applied to basic problems of industrial control. An instrument for measuring and controlling a chemical process with a standard cell was developed and delivered. Devices were constructed for measuring, indicating and recording fluid and air pressures, and accelerometers were designed and fabricated to record continuous or maximum values of acceleration in one or more planes. Photoelectric attachments to safeguard operators of equipment are being investigated for automatically inspecting and rejecting defective products as are devices for grading accurately and sorting or screening out products on the basis of size, color, shape or weight; and for automatically counting and/or recording production runs. The control devices developed have used new designs and adaptations of such components as sensitive relays, saturable core reactors, magnetic fluid clutches, and electronic and magnetic tape controls for predetermined cyclic operations or for functions of automatic equipment. A device was produced for indicating air temperature at all altitudes and at a range of distances from the observer.

(*m*) *Analysis:* The analytical work done has been concerned with fundamental investigations of physical problems and a reduction of resulting expressions to mechanized form. The work included error analysis, approximation theory and the establishment of criteria for conducting tests. Additional work was performed in the fields of automatic control systems. Two projects were started during 1949 involving operations research and human engineering research, thus expanding an important field of activity in which several projects have been successfully undertaken.

(*n*) *Navigation and Communications:* Two systems were designed for navigating and guid-

ing aircraft: the first for pilotless aircraft and the second for traffic control for commercial, private and military aircraft. Two varieties of communication systems were built. The first is a multiplex system for very accurate transmission of factual messages over a pair of wires. The second is a microwave system for angular measurement in all weather.

(*o*) *Aeronautics:* Detailed investigations were made of structural design criteria of piloted and pilotless aircraft and comprehensive reports issued on the subject. This work required examining extensively the physical conditions imposed on surfaces moving at transonic and supersonic speeds and flying at extreme altitudes.

Patents

A procedure was established through which the Institute can extend the privilege of participation in the proceeds of patents in which it may secure rights to employees who contributed importantly to the inventions leading to the patents.

General

It having been apparent that ignorance of the existence of research laboratories at The Franklin Institute remained widespread through industrial circles, special attention was devoted to acquaintance-spreading measures. Nearly 500 persons were conducted on specially planned tours of the Laboratories; additions were made to the technical societies with which joint meetings are held; listing was secured for the Laboratories in additional industrial and laboratory directories; appropriate distribution was made of nearly 5,000 copies of a 40 page illustrated brochure, with copies going to nearly every state in the union and to 36 foreign countries; members of the Laboratories' technical staff continued to publish papers in leading scientific journals, and a planned distribution was made of nearly 400 reprints of them, in addition to honoring the hundreds of requests received for reprints of many of them; and the JOURNAL OF THE FRANKLIN INSTITUTE published each month an expanded abstract of a paper that had been prepared by a member of the Laboratories' staff.

The Bartol Research Foundation

The Bartol Foundation is engaged in work under the following heads:

1. Researches in nuclear physics and cosmic rays,
2. Researches in matters pertaining to thermionic work,
3. Work having to do with the development and construction of a new Van deGraaff generator for positive ions and for a voltage range between 5 and 10 million volts, and
4. Miscellaneous researches sponsored by the Bartol Foundation itself, including the development of a small linear accelerator and researches on solid state physics.

The most significant cosmic ray work during the year has been Dr. M. A. Pomerantz' trip to Churchill, Canada, in which he obtained important information, through cosmic rays, concerning the magnetic field of the sun. Several cosmic ray investigations are also in progress in the laboratory.

The present Van deGraaff generator (2 million volts) continues to function admirably and has been responsible for many researches.

Work on the new Van deGraaff generator has progressed at a rate calculated to fit in with the development of the new building which will house the generator and which, we hope, will be finished in May. The large machine is being sponsored by the Office of Naval Research and by the Bartol Foundation.

Investigations on matters pertaining to solid state physics are being carried on in collaboration with The Franklin Institute Laboratories for Research and Development.

The Biochemical Research Foundation

During 1949 more exact application was made of organic and physico-chemical knowledge to the biological investigations in progress and particularly to the studies in the various phases of cancer research at The Biochemical Research Foundation.

The first of our graduate students who are working on the Foundation's projects and qualifying for the Ph.D. degree at the University of Delaware received his doctorate degree in

September—Dr. George Limperos. His thesis "X-radiation of desoxyribonucleic acid *in vitro* and *in vivo*" received favorable comment in scientific circles and constitutes an important contribution to the Foundation's research. The fact that he is now a member of the Staff is an indication of the mutually derived benefit of the association of the Foundation with the University of Delaware.

The benefits to the Foundation from the research efforts of some of its best members are somewhat counterbalanced by the fact that those Staff members are in constant demand by other institutions. This situation requires careful direction and correlation to maintain an excellent personnel at all times at its best research capacity.

The most important piece of equipment acquired has been the Q-gas Geiger Counter. This apparatus was assembled by our own Staff at a cost considerably less than its equivalent commercial price and functions much more effectively than a factory built one which would not be constructed to meet our special needs.

The publications of the Foundation which appeared in 1949 will be bound together with those of 1948 to form our Volume X of the *Reports of The Biochemical Research Foundation.* These ten volumes, covering 20 years of research, contain 362 publications which had appeared in scientific journals during this period of biochemical research.

Finances

The financial position of The Franklin Institute at December 31, 1949, as compared with December 31, 1948, is indicated in the following comparative summarized statement of assets and liabilities adapted from the report of our auditors Messrs. Lybrand, Ross Brothers and Montgomery.

Assets	*December 31, 1949*	*December 31, 1948*
Current assets	$ 592,305	$ 692,054
Building, equipment, etc.	4,484,218	4,466,598
Less: Reserve for depreciation	(490,272)	(313,943)
Endowment fund investments (including Notes payable contra)	3,917,584	1,711,285
	$8,503,835	$6,555,994
Liabilities		
Notes payable (included in Endowment fund investments contra)	$ 370,000	$ 470,000
Liabilities, deferred income and reserves	161,564	219,785
General and restricted Endowment funds (including, in 1949, Bartol Research Foundation funds)	3,917,584	1,711,285
Capital stock and contributed funds	3,954,346	3,962,514
General appropriation fund (surplus)	100,341	192,410
	$8,503,835	$6,555,994

The results of operation for the year 1949 as compared with 1948 are shown in the following comparative statement of income and expenses:

	1949	*1948*
Operating Revenue:		
Members' dues	$ 56,577	$ 58,067
Museum admissions and other income	95,194	93,590
Research laboratory projects, etc.	1,756,971	1,747,302
Other Institute activities	38,322	34,616
Total operating revenue	1,947,064	1,933,575

Operating Costs and Expenses:		
Museum	271,590	277,815
Library, less amounts transferred	54,566	51,248
Research laboratory projects	1,714,746	1,726,694
Other Institute activities	70,117	66,992
Administrative and general expenses, less amounts transferred	59,349	97,098
Total operating costs and expenses	2,170,368	2,219,847
Operating loss	223,304	286,272
Other Income:		
Income from general endowment fund	70,054	70,314
Portion of income from restricted endowment funds expended during the year	7,237	—
Donations of Friends of Franklin (after temporary transfer of $2,030 to endowment funds in 1949)	20,090	22,150
Appropriation, Commonwealth of Pennsylvania	36,834	34,500
Administrative fees from Bartol and Biochemical Research Foundations	13,639	13,453
Miscellaneous	2,348	1,310
	150,202	141,727
Deficit before other expenses	73,102	144,545
Other Expenses:		
Interest (included in endowment income above)	12,403	12,665
Miscellaneous	5,710	8,577
	18,113	21,242
Net deficit for the period	$ 91,215	$ 165,787

It should be noted that the net deficits shown above are after the inclusion in costs and expenses of charges for depreciation of buildings and equipment amounting to $178,127 in 1949 and $174,843 in 1948.

In recent reports the need of the Institute for unrestricted endowment and gifts has been pointed out. It is with gratitude that we announce the receipt during the year of the unconditional bequest of $5,000 from the late Mr. George W. Childs Drexel; of $2,500 from Mrs. Edward G. Budd and Mr. Edward G. Budd, Jr. to be added to The Edward G. Budd Lecture Fund; and of $23,194 as a specific bequest of $2,000 and one-fourth of the residuary estate of the late Miss Mary Thorn to be added to the Isaac B. Thorn Memorial Library Fund.

The Institute needs funds for the development and enlargement of its Museum educational activities; it needs funds for carrying out self-sponsored research work in its Laboratories; it needs funds for Library books and particularly for the means to make those books more readily available; and of course it needs funds for its general routine, undramatic operations. We believe the Institute should continue to be supported—as it has through 125 years—by the people themselves. We believe this is particularly important in a day when we are seeing on all sides the increasing pressure for authoritarian, centralized state control of a citizen's activities. This tendency would seem to make it increasingly necessary that institutions such as The Franklin Institute stand as bulwarks against such encroachment and serve as centers where educational and scientific activities can be carried on freely without restriction and without

supervision or pressure from governmental authority. This can only be done if those who believe in our work help toward its support to the extent they are able.

Respectfully submitted,
By order of the Board of Managers,
RICHARD T. NALLE,
President

MUSEUM

The significance of paper in our modern civilization is fully recognized by those employed in the papermaking industry. The layman rarely gives it a thought, yet few visitors to the Museum fail to spend some time watching the model Foudrinier machine and, if one may judge from the rapt expression on their faces, they take a keen interest in the subject.

Few people know that Philadelphia has some interesting associations with the papermaking industry in America. The industry had its birth here when William Rittenhouse built the first mill in the year 1690. This fact is generally known, but let us take up the story at that point.

Isaac Langle of Germantown was the first American paper mold maker. Before his time all the molds in which the sheets were made had been imported, but in 1740 the Ephrata Mill in Lancaster County was obtaining molds from him. A quantity of the paper made in these identical molds was used to make cartridges for the soldiers who fought in the battle of Brandywine. By a curious coincidence, one of those soldiers was Nathan Sellers of Upper Darby, whose departure for the army ranks created a technological crisis in the paper trade. Sellers was the only American who could make the wires for the paper molds. The paper trade petitioned Congress to have him discharged. Congress released him, not because the paper trade had made out a good case, but only because Congress was in urgent need of paper money. Without Sellers there would be no molds.

Some time in the 1830's George A. Shyrock of Chambersburg grew weary of collecting linen rags to make paper and tried straw. This was such a success that his example was followed by other paper makers. Martin Nixon of Manyunk made some of the straw paper for the *Evening Public Ledger*, which had the distinction of being the first newspaper printed on anything but paper made from linen rags. It could hardly be called a success, judging from the impolite remarks addressed to the publishers of the yellow sheets. Complaints of nondelivery were eventually traced to the goats of North Philadelphia which were greedily eating every paper that had been left on doorsteps. The historian who reports this episode offers excuses for the paper, saying that the high mortality among the goats was to be attributed to the poor quality of the ink used by the publishers.

When the time came for experiments among wood pulp, Philadelphians were in the forefront. In the 1860's Hugh Burgess and Morris Keen set up a soda wood pilot plant in the old engine shed on the bank of the Schuylkyll River at the end of Gray's Ferry Road. Here they boiled wood in caustic soda at a high temperature. They met with success, and later established extensive mills at Royersford and Manyunk which made Philadelphia famous again as a papermaking center.

The present method of making wood pulp is often traced to the process of Fry and Ekman in Sweden, but their work was merely a continuation of that done by Benjamin Tilgham in Philadelphia. It was Tilgham's ill-fortune to complete his process during the last year of the Civil War, when no one was willing to finance a new industry, so he abandoned it to the two Swedes.

These local associations should be remembered when watching the model Foudrinier papermaking machine at work in the Museum. It is not often that one has the opportunity to see one of these machines in actual productive use, or to see it all at one time. This makes the little model of especial interest. It is the smallest scale model in operation, and the visitor has the entire process under observation from pulp to finished paper. Some of this paper is used in Institute activities, but schoolboys are the principal customers.

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, June 14, 1950.)

HALL OF THE COMMITTEE,
PHILADELPHIA, JUNE 14, 1950.

MR. LEWIS P. TABOR in the Chair.

The following reports were presented for final action:

No. 3218: Clamer Medal.

This report recommended the award of the Francis J. Clamer Medal to Charles Sanborn Barrett, of Chicago, Illinois, "In recognition of his outstanding contribution to the knowledge of crystal structures and of the effects of deformation and aging on various metallic crystals."

No. 3227: Brown Medal.

This report recommended the award of a Frank P. Brown Medal each to Eugène Freyssinet, of Paris, France, and Gustave Magnel, of Ghent, Belgium, "For their outstanding development of engineering and technique for prestressed concrete members resulting in radical improvements in the design and construction of bridges, buildings and other structures."

JOHN FRAZER,
Secretary to Committee.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS June 21, 1950

HONORARY

Eugene Paul Wigner, D.Sc.

ACTIVE FAMILY

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ACTIVE NON-RESIDENT

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NECROLOGY

William D. Disston '13	Norman Foster '46	Robert F. Jaegle '42
W. Kirkland Dwier '36	Charles Hart '17	D. Newton Miller '45
	William Albert Hyde '35	

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

Random-Circuit Theory of Resistance and Noise in a Composition Resistor,[1] by ROBERT P. COLEMAN.[2] Common types of resistor are made from a mixture of a finely divided conducting material, such as carbon, in a non-conducting binder. As the resistance of such a mixture may be immensely greater than that of the pure conducting material, it is reasonable to attribute the high resistance to the imperfect contacts between conducting particles. Such a heterogeneous conducting medium has electrical properties that may be analyzed in terms of the concept of a random electrical circuit. In this conceptual model, the imperfect contacts between conducting particles constitute the resistance elements, and the particles themselves constitute the connecting leads between resistance elements.

There are three problems, or fields of research, involved in accounting for the resistance properties of a composition resistor, and ultimately it is necessary to solve all three in order to account for the electrical quantities measured at terminals of a resistor. These problems are: (1) The fundamental mechanism of conduction phenomena in local regions; (2) The statistical distribution of local resistance values as determined by such things as the concentration, particle size, and surface properties of the constituents; and (3) The relation between the statistical distribution of local resistance values and the resistance measured at the terminals of the resistor.

The present paper deals only with the third problem.

We shall treat the case of a thin film considered as a single layer of particles. The resistor is thought of as an electric circuit consisting of local resistance elements, $r_1 \cdots r_N$, arranged in a random manner within a rectangular area and with adjacent elements connected. Terminals are assumed to be connected to opposite edges of the rectangular film which is of length l and width w. The terminal resistance R is then a function of all the local resistances $r_1, r_2, \cdots r_N$. The form of this function is obtained from the expression for power which can be written either in terms of the terminal resistance and current or as a summation of the power in each local resistance. The relation between the local currents and the terminal current is obtained from a study of the circuit in which attention is fixed upon one typical local resistance together with parameters that depend upon the average effect of all the other resistors considered to be randomly distributed. Introduction of a characteristic length l_1 related to the "grain-size" and of an average resistivity ρ leads to a resistance equation of the form

$$\rho = \frac{l_1^2}{lw} \sum_{k=1}^{N} \left(\frac{1}{1 + \frac{r_k}{\rho}} \right)^2 r_k.$$

The problem is now reduced to that of finding a self-consistent value of ρ in this equation. In doing this, the value of l_1 is adjusted so that only one value of ρ satisfies the equation. A few solutions of this equation have been investigated under rather general assumptions about the distribution of values of r_k.

Other electrical properties can also be treated by an extension of the foregoing analysis. For example, an upper bound to resistance-fluctuation noise can be found under the assumption that noise can be represented by independent random fluctuations in r_k.

The foregoing analysis suggests that fine grain size and narrow distribution of local resistance values, r_k, are desirable to reduce noise at the terminals of a heterogeneous resistor.

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[1] This work was presented at the Oak Ridge Meeting of the American Physical Society, March 16–18, 1950, and is a sequel to some work that was supported by the International Resistance Co. at The Franklin Institute.

[2] Chief of Solid State Physics Section, The Franklin Institute Laboratories, Philadelphia, Pa.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

MERIAM, J. L.: Differential Analyzer Solution for the Stresses in a Rotating Bell-Shaped Shell.

MOON, PARRY AND DOMINA EBERLE SPENCER: Interflections in Coupled Enclosures.

DANFORTH, W. E.: Radiation Heating of a Plane Parallel Slab by a Parallel Semi-Infinite Heater.

DANFORTH, W. E., AND T. A. HADDAD: Radiation Transfer Considerations in the Heating of a Cathode Sleeve.

BORG, S. F.: Additional Interpretations of the Solution of the Straight Beam Differential Equation.

SWANN, W. F. G.: On Certain Matters Pertaining to Electrets.

AYRE, ROBERT S., LYDIK JACOBSEN AND ARIS PHILLIPS: Steady Forced Vibration of a Non-Conservative System with Variable Mass; a Pumping System.

KRZYWOBLOCKI, M. Z.: On the Asymptotic Expansion in Three-Dimensional Compressible Viscous Flow.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

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RECENT ADDITIONS

AGRICULTURE

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HENDERSON, JAMES LLOYD AND ROADHOUSE, CHESTER L. The Market Milk Industry. Ed. 2. 1950.

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BARTOL RESEARCH FOUNDATION

Characteristic Radiations of Rhodium (105)*, by C. E. MANDEVILLE AND E. SHAPIRO. The 36.8-hr. rhodium (105) was grown from its short-lived ruthenium parent when metallic ruthenium was irradiated by neutrons in the Oak Ridge pile. The target material was purified prior to irradiation by the distillation of RuO_4 from a perchloric-sulfuric acid solution of $RuCl_3$. Chemical separations for the removal of ruthenium and technetium were carried out subsequent to exposure. The beta rays have a maximum energy of ~ 0.6 Mev as measured by aluminum absorption, and the gamma rays were found to have an energy of 0.30 Mev when absorbed in lead. The beta-gamma coincidence rate is independent of the beta ray energy, and the magnitude suggests that the harder beta rays are coincident with soft quanta of the x-ray region. Very few beta rays are coincident with the gamma ray at 0.30 Mev. No beta-beta coincidences were detected for ranges exceeding 6 mg./cm.^{2} No gamma-gamma coincidences were found.

* Assisted by the joint program of the ONR and the AEC.

Radiations from Cerium (144) $\xrightarrow{\beta}$ Praseodymium (144) $\xrightarrow{\beta}$ Neodymium (144)*, by C. E. MANDEVILLE AND E. SHAPIRO. Ce^{144}, in equilibrium with its 17-min. daughter activity, Pr^{144}, was separated from fission fragments at the Oak Ridge pile. After further chemical purification, coincidence, and absorption measurements were made of the emitted radiations. The beta-rays of the 300-day Ce^{144} were found to have a maximum energy of 0.36 Mev, and the very hard beta-rays of Pr^{144} have an end point at 2.87 Mev. A source of the two radionuclides in equilibrium emits X-rays at 36 kev and gamma-rays having a mean energy of 1.67 Mev as determined by absorption curves in lead and aluminum. A coincidence absorption curve indicated that a hard gamma-ray at 2.60×0.15 Mev is emitted in the disintegration of Pr^{144}. The beta-gamma-coincidence rate of Pr^{144} was observed to decrease from an extrapolated value of 0.02×10^{-3} coincidence per beta-ray at zero absorber thickness to zero at an approximate beta-ray energy of 0.42 Mev. The magnitude of the coincidence rate and the calibration of the gamma-ray counter indicate that two per cent of the beta-rays of Pr^{144} are coupled with the hard gamma-radiation. Gamma-gamma- but no beta-beta-coincidences were found in Ce^{144} — Pr^{144}.

* Assisted by the joint program of the ONR and the AEC.

Optimum Parameters for Particle Acceleration by TM 010 Cylindrical Cavities, by B. L. MILLER. Several linear accelerator projects made use of cylindrical cavities operating in the

TM 010 mode. The energy imparted to the beam is somewhat increased by using less than a full half-cycle, because in the shorter cavity a higher field is attained for the same power input. For particles injected at velocity c, the cavity length for maximum acceleration is 0.44λ and the optimum injection angle 11° after the zero field phase, independent of the r-f power level.[1] For initial velocities less than c, the optimum cavity length and injection angle depend both on the injection velocity and the r-f power available. Calculations of these optimum parameters have been made in the low energy case, for small relativistic correction, and in the far relativistic region where most of the orbit is traversed near velocity c. Because the optimum values do not vary widely (for example, injection angle always less than 11°), they can be estimated, from the above calculations, for the intermediate region.

[1] W. GRAFFUNDER, *Helv. Phys. Acta*, Vol. 22, p. 239 (1949).

Nuclear Energy Levels in Nitrogen (14) and Aluminum (27),* by C. P. SWANN AND C. E. MANDEVILLE. Ilford C_2 emulsions, making an angle of zero degrees with the bombarding beam, were irradiated by neutrons from the reaction $C^{13}(D,n)N^{14}$. The incident deuterons were supplied with an energy of 1.43 Mev by the Bartol Van de Graaff statitron. The target of C^{13} (isotopic concentration 50 per cent) had a thickness of 100 kev. Neutron groups were observed corresponding to Q-values of 0.27, 1.30, 1.68 and 5.24 Mev giving energy levels in N^{14} at 4.97, 3.94, and 3.56 Mev. The group of lowest Q has been previously reported by the Rice group.[1] The intermediate Q-values offer a possible explanation for recently observed gamma-rays.[2] When a thick target of isotopic Mg^{26} was irradiated by deuterons, eight groups of neutrons were observed corresponding to energy levels in Al^{27} at 0.88, 1.92, 2.75, 3.65, 4.33, 5.32, and 5.81 Mev. The Q-value for the ground state is 5.68 Mev. The level structure closely resembles that obtained by other methods; for example, proton scattering by aluminum.

* Assisted by the joint program of the ONR and the AEC.
[1] BENNETT, BONNER, HUDSPETH, RICHARDS, AND WATT; *Phys. Rev.*, Vol. 59, p. 781 (1941).
[2] R. G. THOMAS AND T. LAURITSEN, *Bull. Am. Phys. Soc.*, Vol. 24, No. 8, p. 11 (1949).

On Certain Matters Pertaining to Electrets, by W. F. G. SWANN. Consider an electret of thickness L with surfaces perpendicular to the axis of x at $x = 0$ and $x = L$, and polarized in the positive direction of that axis. Suppose that P_x decays with time through a factor $e^{-\alpha t}$ and that initially, as the result of the polarization and surface or volume charges, the potentials at $x = 0$ and $x = L$ are V_0 and zero. Suppose the surfaces covered by foils joined by a resistance R which may be the internal resistance of the dielectric. Let C be the capacity. It results that if the disappearance of the charges is determined by Ohm's law, the variation of the potential V at $x = 0$ *will be independent of the original distribution of charge.* Provided that V_0 is negative, V will drop to zero in a time τ_1 of the order of RC. It will then rise to a positive maximum V_m in a time τ_2 and will finally decay to zero. Reasonable values of the quantities concerned are consistent with τ_1 being comparable with a day or two, τ_2 with two weeks, V_m of the order of 2000 volts with a decay to $\frac{1}{2}$ of this value in three years.

BOOK REVIEWS

JOSEPH HENRY: HIS LIFE AND WORK, by Thomas Coulson. 352 pages, 16 × 24 cm. Princeton, Princeton University Press, 1950. Price, $5.00.

The only sources of information hitherto available about Joseph Henry, probably the greatest 19th-century American scientist, consisted of a number of scattered magazine articles, the "Memorial" collected after his death and published by order of Congress in 1880, and short accounts in various histories of science, among which Crowther's seventy-page narrative (in *Famous American Men of Science*) is the most complete. The task of writing a full-scale biography of Henry has thus long been incumbent upon American historians: in this task, Mr. Coulson has succeeded most admirably.

Joseph Henry died in 1878 at the ripe age of eighty-one, after a lifetime of unflagging industry and remarkable achievement—but the discoveries which brought him international fame were made largely before he was thirty-five. He was a teacher at the Albany Academy when he discovered mutual induction (contemporaneously with Faraday, who published his results first and is consequently acknowledged as the discoverer); and he was still at Albany when he announced his greatest single contribution to science, the discovery of self-induction. It is an interesting commentary on the limited circulation of American scientific journals in Europe at this time that Faraday was apparently not aware of Henry's work, for two years later the British scientist announced his observation of the phenomenon of self-induction as an original discovery. But world-wide recognition came to Henry before long, and when the International Congress of Electricians met in Chicago in 1893, the suggestion of two foreign representatives (from France and Britain) to give the name "henry" to the standard unit of inductance was adopted unanimously.

Henry's great early contributions have tended to overshadow his later work. Mr. Coulson points out that Henry developed the electromagnetic relay, drew attention to the utility of the transformer, investigated the effects of electromagnetic shielding, and was the first to establish the oscillatory nature of an electric discharge through an inductance (which is the basis for radio transmission). After he was appointed to a professorship at Princeton, he also studied the phenomena of capillarity, phosphorescence, and atmospheric electricity. He made important contributions to the science of meteorology and started an investigation of the correlation of natural forces which led him to the very brink of the concept of conservation of energy. He was at the peak of scientific endeavor when the offer to become secretary of the newly formed Smithsonian Institution was brought to him. He considered the proposal briefly, not without misgivings, and decided that he could serve the cause of American science best by accepting the post—realizing all the while that he was, as he himself put it, "sacrificing future fame to present reputation." The thirty-two years of his secretaryship saw the Smithsonian rise from a vague clause in an Englishman's will to a world-famous center of scientific activity and clearinghouse of scientific information. The present reputation of the Institution is perhaps the best memorial to Joseph Henry: *si monumentum requiris, circumspice.*

Mr. Coulson, who is director of museum research at The Franklin Institute, has been able to uncover a good deal of new material through his study of the manuscript resources of the Smithsonian, and he has done an excellent job of bringing together the scattered scraps of information previously available. He has not hesitated to point out some of the limitations of Henry, who was largely an experimental scientist and, in Mr. Coulson's words, "probably the last physicist to attain international fame by the use of simple arithmetic as his sole mathematical tool." The author has nowhere allowed (except perhaps in his account of the Morse-Henry telegraph controversy) to let his warm partisanship of Joseph Henry interfere with complete detachment and objectivity. The book is pleasantly written and well-documented, and represents an important contribution to the history of science.

CHARLES SUSSKIND
Yale University

GEOMETRICAL CONSTRUCTIONS WITH A RULER, GIVEN A FIXED CIRCLE WITH ITS CENTER, by Jacob Steiner. Translated from the first German edition (1833) by Marion Elizabeth Stark, edited by Raymond Clare Archibald. 87 pages, diagrams, port., 17 × 25 cm. New York, Scripta Mathematica, 1950. Price, $2.00.

This small book is the first English translation of any work of Jacob Steiner. It treats the subject of geometrical constructions with a ruler, given a fixed circle with its center. Steiner's aim was to show that any point which can be constructed with ruler and compasses can be constructed with the ruler alone, if a circle and its center are given in the plane of construction. Chapter headings are: I. Some Properties of Rectilinear Figures, and Resulting Constructions with Ruler Alone; II. Some Properties of the Circle; III. Solution of All Geometrical Problems by Means of the Ruler, when a Fixed Circle and Its Center Are Given; and Appendix, Miscellaneous Problems.

Though elementary in nature and insufficient in itself for a text in synthetic geometry, it can serve teachers as an English text for supplementary reading assignments in this subject. It also offers those interested in the history of mathematics insight into one of the earliest works in this field. The book is carefully translated and amply fortified with a large set of notes including many references.

D. B. HOUGHTON

FUNDAMENTALS OF DETERGENCY, by William W. Niven, Jr. 256 pages, illustrations, 18 × 25 cm. New York, Reinhold Publishing Co., 1950. Price, $5.50.

As indicated by the title, this book deals mainly with general principles, and treats individual compounds only to a minor extent. Unfortunately, no one knows just what are the fundamental principles of detergency. The author includes all the properties of detergent solutions that might reasonably be expected to be involved, and perhaps a few others. The nature of solutions of soap and other detergents is discussed, followed by the various physical properties of the solutions. Builders and their effects on the detergent solutions are then described. The second section of the book attempts to show how these principles may be applied to the cleaning of cloth. There are many graphs and schematic diagrams of detergent molecules.

The main value of the book lies in the collection of physical data: percentage hydrolysis, surface tension, interfacial tension, etc. The discussions and explanations are interesting, but the conclusions must not be taken too seriously. In many cases recent developments and conclusions are neglected. The author is also guilty of helping to perpetuate such errors as the name "hexametaphosphate."

Obviously a book of this size cannot include all of the rapidly expanding field of detergency. Only a few general types of synthetic detergents are mentioned. The treatment of builders is hardly adequate. Methods of measuring detergency are not described.

The book is a valuable contribution to the subject, filling an important need with its collection of data and literature references. But the reader must beware of out-of-date theories and explanations.

NEWTON W. MCCREADY

INTRODUCTION TO SEMI-MICRO QUALITATIVE CHEMICAL ANALYSIS, by Louis J. Curtman. Revised edition, 391 pages, illustrations, plate, 14 × 21 cm. New York, The Macmillan Co., 1950. Price, $3.50.

This, the first revision of Dr. Curtman's book, provides an authoritative and up to date course in semi-micro qualitative analysis. The changes to be found in this edition are primarily in the procedures for analysis. The author together with his research workers have both modified and perfected the analysis schemes wherever they found it necessary. The other revisions are found in the appendix where a system for checking in and out as well as a list of student assignments and lecture topics is included.

The subject matter of this volume is divided into five main parts plus an introduction and appendix. The introduction apprises the student of the methods, terms, and limitations of qualitative analysis. Part one contains the theories necessary for an understanding of the sub-

ject, for example, the theories of chemical and physical equilibrium, of solubility products, of redox reactions, and the theory of solutions. The second section gives the equations for the reactions used in the detection of the cations and anions. Part three explains calculations particularly valuable in analytical chemistry. Part four deals with laboratory work and includes some preliminary tests for analyzing known samples in order to perfect the workers technique and familiarize him with the reagents. The fifth section provides schemes for the systematic detection of both cations and anions in the presence of interfering ions on unknown samples. The appendix contains such information as desk equipment, reagents, directions for preparing stock solutions, and such data as are useful in calculating the problems found in analytical chemistry.

This book is meant to serve as both a laboratory manual and text book for a full year course in qualitative analysis. The choice of problems is very good for they are directly applicable to the laboratory work and are sufficiently varied to provide the student with a complete understanding of the principles involved. The material in the section devoted to theory is particularly impressive in its completeness, and it provides a much needed foundation for the advanced work of quantitative analysis and physical chemistry. The reviewer feels that one improvement could be made by putting the flow sheets in bold type and making them full page size. The author most certainly accomplishes his purpose of closely coordinating theory and laboratory work. Curtman's book is excellent and stands as one of the best in its field.

DONALD H. RUSSELL

QUALITY CONTROL AND STATISTICAL METHODS, by Edward M. Schrock. 213 pages, diagrams, 15 × 24 cm. New York, Reinhold Publishing Co., 1950. Price, $5.00.

Quoting from the author's preface, "The purpose of this book is to present as clearly as possible, as briefly as practicable, and yet adequately, the most generally useful modern techniques of quality control and statistical methods as applied in industrial problems of product quality. It is intended for those who are new to the field (or relatively so) and want to improve their effectiveness in appraising and controlling (or helping to bring about control of) quality."

The mode of presentation and the scope of the subject covered in the text are rather more designed for the parenthetically defined class of readers and the book appears to have grown out of a training course for the quality-control personnel of the General Electric Company with which the author is associated.

An introductory and the first two chapters serve to drive home, in question-answer style and falling back on case histories, the economic advantages of quality control to the novice and his recalcitrant boss.

The principal part of the book is devoted to the description of statistical methods of securing and processing information about the industrial process to be controlled, culminating in a quality-control chart for this process. Variants of these procedures best suited for different types of control problems are discussed at considerable length. In most arguments, a normal (Gaussian) frequency distribution of data is assumed to exist, but other types of distribution are also briefly considered.

The last four chapters deal with the question of significance of differences in statistical parameters; the problems of acceptance sampling; the method of sequential analysis; and the use of the least-squares method in the correlation of scattered data.

Numerous typical graphs, as well as charts and tables representing functional relations among various statistical parameters contribute to the general usefulness of the book.

A glossary, a subject index and a bibliography of ten texts on statistical methods in industry are appended.

This reviewer, who does not fall into the class of readers contemplated by the author, yet is familiar with problems of industrial production and with applications of statistical theories in research, is not wholly pleased with the manner in which the material is presented. No serious attempt is made to outline the concepts underlying the statistical techniques, so that the text is, in effect, an enumeration of "cook-book recipes" for the preparation and use of quality-control charts. Too little attention is paid to the definitions of statistical and production terms used in the text, which far outnumber those compiled in the glossary. The typo-

graphical layout of Table 9.7a (pages 68–71) is unnecessarily confusing; the charts (Figs. 9.7a and 9.7b) ought to have been printed on one side only of two separate sheets. This table and charts as well as Table 9.7b (pages 73–122) should have been relegated to an appendix.

In this diffuse matrix are embedded comments, practical hints, and general observations stemming from the author's practical experience in the field of quality control, which will prove to be of considerable value to the instructor of an industrial course in this subject.

In toto, the organization of the book is an excellent example of a system *in a state of statistical control*, in which, quoting the author, "The variations of successive observations of the quality characteristics involved are due solely to a constant system of chance causes. It will be noted that there is no suggestion here that anything happens just 'by accident' or without any cause whatsoever. It is definitely stated that causes of quality variation are at work, but identifies them as 'chance causes'; that is, they are permitted to vary as they may without any effort being made to control or eliminate them or even to identify them. Individually, their effect on quality variation is negligible; collectively they make up what is known as *inherent variability*."

OTTO R. SPIES

BOOK NOTES

A SYSTEMATIC LABORATORY COURSE IN GENERAL CHEMISTRY, by Harry H. Sisler and Jay J. Stewart. 396 pages, illustrations, tables, 21 × 28 cm. New York, The Macmillan Co., 1950. Price, $3.00.

Employing perforated and punched pages to facilitate turning in work for checking and subsequent return to the student's notebook, this manual presents ninety experiments in general chemistry. The authors have attempted to direct the student to interpret his observations in terms of the currently accepted theories of atomic and molecular structures.

ELEMENTS OF PRACTICAL AERODYNAMICS, by Bradley Jones. Fourth edition, 444 pages, illustrations, diagrams, 14 × 21 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

In revising this elementary survey of aerodynamics, the order of several of the chapters has been changed to improve the continuity of the treatment. The number of illustrative problems and those for exercise have both been increased. Some descriptive material not especially pertinent to aerodynamics has been omitted.

THE AMERICAN ECONOMIC SYSTEM, by Frank D. Newbury. 558 pages, tables, graphs, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $5.00.

Intended as a textbook to promote an understanding of the economic fundamentals underlying current problems and to show the application of these principles to practical situations. The American economic system has been presented in detail—what it is, how it works, and what it has achieved. Thus emphasis has been placed on the operation of the actual dynamic system and static theory has been reduced to a minimum. In so far as possible, realistic data from industry have been chosen with a view to emphasizing the controversial nature of many economic situations.

CHEMICAL ENGINEERS' HANDBOOK, prepared by a staff of specialists with John H. Perry, editor. Third edition, 1942, pages, illustrations, tables, graphs, 18 × 26 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $15.00.

In this third edition of Perry's handbook, a new format has been adopted. The larger page size reduces the bulk of the book, at the same time permitting the use of larger sizes of graphs and illustrations where it has been desirable. The considerable progress in chemical engineering during the war years has necessitated the revision of nearly all sections, many of which have been rewritten and expanded as well. New sections have been added on the General Theory of Diffusional Operations; Furnaces and Kilns; and Size Enlargement. Of necessity, some material has been deleted to permit the addition of what seemed more valuable.

ERRATUM: The review of EARTH WAVES, p. 340 of this JOURNAL for April, 1950, listed the author as I. Don Leet, instead of L. Don Leet. Also, the book is a joint publication of the Harvard University Press and of John Wiley & Sons, Inc.

Two New Electrostatic Generators Developed for Army Engineers (*Technical Data Digest*, Vol. 15, No. 4.).—Two electrostatic generators, developing 20,000 v. and 6000 v. respectively, have been built for the Army's Engineers Research and Development Laboratories at Fort Belvoir, Va. The spring-driven devices weigh only about 10 lb. each, and are designed for use with certain pieces of portable military equipment.

With a gear reduction of 14 to 1 between the spring motor and the generator, and a variable governor, the generators can operate up to 19 min. on a single winding. Electrostatic generators differ from electromagnetic generators in that speed determines current in the former, while it determines output voltage in the latter.

Research toward the development of high potential voltages generating small direct currents at high voltages was needed in the application of electron picture tubes. The new influence-type generators were built by the Chatham Electronics Corp., Newark, N. J.

They were made possible by a combination of new insulation materials, new plastics, and new techniques of hermetic sealing. The use of a "pressure tight transmission" allowed the application of a rotary motion into the pressurized generator mechanism. Although light in weight, these generators are believed to be the first models for picture-tube application, and ERDL scientists think further size and weight reductions are possible.

Excellent voltage regulation has been obtained by using corona-discharge regulation tubes, another new development incorporated in the devices. Acting as a kind of safety valve, these constant and variable regulators limit, and thus maintain, specified voltages. Use of these tubes has eliminated power-consuming, resistance-type voltage dividers, and, with them, the need for high input currents.

The generators consist of a spring motor driving a rotor, made up of plastic laminated with metallic foil, between charged insulated metal plates. As the rotor turns between the charged plates, a voltage is built up by induction on the metallic foil.

At the proper time, the voltage is picked off the foil by means of brushes and transported to the insulated metal plates. After these plates are initially weakly charged by a friction-type electrostatic generator, the process of building up a voltage and transporting it to plates is continuous as long as the rotor is in motion. The charges accumulated on these plates are the source of the high voltage.

A friction-type electrostatic generator is needed to supply the initial charge to the insulated metal plates of the influence-type generator for starting. It develops its potential by the process of charging a body by rubbing it with another material. This phenomenon may be explained by the concept of transfer of electrons at the interface of the two materials, where electrons are attracted to that material which is more electropositive.

***Abstract of* Clostridia in Gas Gangrene.**—L. DES. SMITH (*Bact. Revs.*, **13**:233, (1949)). This review of the literature is restricted, largely, to the investigations carried out since the beginning of the second World War, and is primarily concerned with the factors affecting the growth of the anaerobic spore-forming bacteria in the animal body. In spite of the advances that were made in medicine in the period between the first and second World Wars, the incidence and mortality of gas gangrene was as high in the last war as in the previous one.

In general, the incidence of gas gangrene in wounded men is appreciable only in troops engaged in ground warfare, being rare, indeed, in naval and air force personnel. This readily follows from the fact that the soil is the major natural habitat of the pathogenic clostridia, and from the fact that the infantry are the most intimately associated with mud in the wet season and dust in the dry.

The type of wound inflicted by modern high-speed missiles is well suited to the growth of anaerobic bacteria, for there usually is an appreciable amount of tissue damage. When a small missile strikes a heavily muscled portion of the body such as the thigh, it makes a small hole in the skin, a larger hole in the subcutaneous fat and then pierces the inelastic deep fascia surrounding the muscle which responds by active reflex contraction, pulling the damaged fibers widely apart with the formation of a relatively large cavity. During this period, much of the kinetic energy of the missile is transformed into shock wave pressures. Regions of very high pressure are formed immediately in front and to each side of the moving missile. Relatively slow low pressure changes are connected with the behavior of the explosive temporary cavity formed behind the missile. The primary damage to the tissue is due to the crushing of the tissue in front of the missile and the stretching or tearing around the missile path which results from the formation and collapse of the temporary cavity. Localized and sharply defined areas of anoxic muscle are produced, probably as the result of local vascular spasm, and regions of extravasated blood are formed.

It is in the damaged and anoxic muscle, or within the regions of extravasated blood that the clostridia find conditions suitable for growth. Once these organisms have started to grow, further local damage to the tissues is inflicted by the exocellular enzymes secreted by these bacteria, for it is only by the action of these enzymes on the tissue that the bacteria can obtain the substances necessary for their multiplication. For example, *Clostridium perfringens* utilizes as one of its principal sources of energy the muscle glycogen which is rendered available by the glycogen-splitting enzyme secreted by this organism. The nucleic acids of the host tissues are split by ribonuclease and perhaps desoxyribonuclease, which serve to supply the invading bacteria with adenine and uracil,

while the hyaluronidase supplies it with N-acetyl-glucosamine. Amino acids and peptides are probably supplied primarily by collagenase, an enzyme which also serves to break down the architecture of the muscle groups through destruction of connective tissue. The lecithinase serves to disrupt the cellular elements of the tissues through the destruction of cell membranes, thus not only destroying the phagocytic cells, but also making accessible to the other enzymes substrates which normally would be protected by the cell membrane. The fermentation of glycogen and amino acids serves to result in the accumulation of hydrogen and carbon dioxide within the inelastic sheath of connective tissue surrounding the muscle, and by thus increasing the pressure locally, impairs the blood supply to the affected area. This impairment of blood supply results in a lowered oxidation-reduction potential, which is necessary for the growth of the invading bacteria and the spread of the infection.

In spite of the advances in chemotherapy that have been made in recent years, it appears that no chemotherapeutic agent has been found which markedly affects the progress of clostridial infections of man. The sulfonamides have no appreciable effect, and among the antibiotics, penicillin has not been found to be of much value. Antibiotics other than penicillin that have been tried—notatin, gliotoxin, gramicidin, tyrothricin, streptothricin, streptomycin—were less effective in laboratory animals than penicillin. Oxidizing agents such as zinc peroxide, urea peroxide, and pyrophosphate peroxide, were also ineffective. It appears, then, that the growth of the clostridia within the body is beyond our control so far as chemotherapy is concerned. The only measures available to the surgeon are total excision of the invaded muscle, where this is possible, and the liberal use of antitoxin—the methods used during the first World War.

***Abstract of* The Anomalous Heat Inactivation of *Clostridium perfringens* Lecithinase.**—L. DES. SMITH AND M. V. GARDNER (*Arch. Biochem.*, **25**:54 (1950)). The lecithin-splitting activity of culture filtrates of *C. perfringens* was found to be more greatly inactivated by heating at 60 to 70°C. than it was by heating at 100°C. When material which had been inactivated by heating at 65°C. was heated for a few minutes at 100°C. partial reactivation occurred. Crude lecithinase preparations were partially purified by electrodialysis or precipitation with nucleic acid. After this treatment they no longer exhibited anomalous heat inactivation. This property could be conferred on them, however, by the addition of Ca^{++} or $Mg.^{++}$ Lecithinase which had been inactivated by heating at 65°C. in the presence of Ca^{++} was centrifuged out of solution, and reactivated by heating at 100°C. After this second heating, the lecithinase was no longer centrifugable. It was concluded that lecithinase in the presence of Ca^{++} or Mg^{++} formed complexes when heated at 65°C., and that these complexes were disaggregated when heated at 100°C. by the formation of an insoluble calcium compound, probably one of the phosphates.

CURRENT TOPICS

Quik-Shot Soldering Irons.—The John F. Rider Laboratories, Inc., have taken over the national sales distribution in the United States and Canada of the Quik-Shot Soldering Iron, which requires no electricity or flame to heat it to a soldering temperature. The necessary heat is developed internally by means of a patented chemical content within the Quik-Shot cartridge, which is inserted into the iron and "fired" by means of a spring operated trigger. It reaches soldering temperature in a matter of seconds and present irons maintain soldering temperature for about eight minutes—more than an ample amount of time to satisfy emergency needs.

The Iron was conceived to take care of the myriad emergencies which develop outdoors and indoors when soldering must be done and electrical power is not available, or when access to line power is not convenient, and other means of heating soldering irons are hazardous to life and property.

Men installing or maintaining TV or amateur radio antennas, radio equipment in mobile vehicles, loudspeakers and amplifiers on towers or stadiums or drive-in theatres, mobile sound trucks, private boats and planes, power line and telephone crews, will find the Quik-Shot Soldering Iron indispensable. It is the emergency soldering iron for one thousand and one important uses.

The Quik-Shot Soldering Iron is presently being used in many industries such as electronic, oil, electrical, marine, automotive, and others, every place where safe and efficient soldering must be done to meet an emergency need.

Alertness Indicator (*Technical Data Digest*, Vol. 15, No. 5.).—A portable "alertness indicator," which signals dangerous or undesirable lethargy in performing monotonous routine tasks of long duration, has been designed by the Laboratory of Sensory Psychology and Physiology of Tufts College, Medford, Mass.

Developed under contract with the Navy's Special Devices Center, Sands Point, L. I., and in cooperation with other branches of the Armed Forces, the device has been tested in automobiles, trucks, and aircraft.

The alertness indicator consists of a group of electronic instruments and recording devices housed in two units, each about one cubic foot in size. Electronic pickups, strapped to the head of a person performing a task, relay "muscle spikes" from facial muscles in the region of the forehead to the indicator.

The readings obtained give an indication of any rise or fall of alertness experienced by the driver of a vehicle, a pilot, or any other person performing a monotonous task, over an extended period of time.

According to Tufts College researchers, some important information has been obtained from tests of a truck driver, operating a large truck and trailer from New York City to Boston, Mass.

During this run, the alertness indicator was placed on the seat between the driver and the observer. Readings were taken every 5 min. over a period of 11 hr. actual driving time.

Results showed that several situations gave rise to a relatively high degree of alertness, while other situations caused leveling off or a substantial drop in alertness.

In terms of the electrical output from the driver, the following tabulated data on the driving conditions and accompanying alertness indicator readings show the differences between easy and difficult driving conditions:

Driving Condition	Electrical Output From Forehead of Driver
Turning trailer around	10 μv
Heavy traffic	9.1 "
Sharp curves	9 "
Sudden stop	9 "
Sudden speeding up	8 "
Shifting gears	7.2 "
Passing vehicles	7.2 "
Driving at high speeds	7 "
Rain, snow	6 "
Intersections	6 "
Easy driving along straightaway	3.5 "
Good driving conditions	3.5 "
Stopping and waiting at traffic lights	2 "
During ferry trip	1.2 "

These data indicate great differences in energy demands on the driver while driving under various conditions of traffic and weather.

Another field test of the alertness indicator was made by Tufts College engineers on board the C-47 Airborne Psychology Laboratory of the Aero Medical Lab at Air Materiel Command Headquarters. The flight was made over a period of 17 hr. in the air, with one pilot continuously at the controls.

A series of ten alertness indicator readings was taken every 15 min. during the flight. These readings showed: (1) A decline of alertness shortly after the task had been started. (2) A long period during which tension level remained above the alertness level. (3) A period near the end of the task when the subject's alertness level fell to the point of operating the warning signal, recovered, fell again, recovered, etc. (4) An end spurt shortly before the end of the task, during which the pilot's tension level stayed well above the unalertness level.

It was found that of the ten times the alertness indicator signaled declined alertness, eight of these points occurred during the last four hours in the air and five of these occurred in very close succession.

The data obtained from the various tests conducted by Tufts College indicate that with the use of proper warning signal, which is easily attached to the alertness indicator, the operator performing a task will know when he is becoming tired or dangerously unalert.

Further tests of the alertness indicator are planned aboard a submarine.

Future plans call for reducing the size of the unit to one cubic foot, without increasing interference among parts of the circuit. Rugged construction will still be maintained.

Among the other problems which Tufts College researchers hope to solve with the device are: (1) What are the amplitude and frequency characteristics of the electrical output from the forehead region? (2) Does this output vary from subject to subject? (3) Does this output vary from occupation to occupation with the same subject? (4) What is the real origin of this bioelectric output?

Examining specimen on metallographic microscope at Bell Telephone Laboratories.

The look that keeps telephone costs DOWN

Through his microscope this Bell metallurgist examines a bit of material which is proposed for telephone use. From what he sees of grain structure, he gains insight into performance not provided by spectrum or chemical analysis. He learns how to make telephone parts stand up longer, so that telephone costs can be kept as low as possible.

The items which come under scrutiny are many and varied, ranging from manhole covers to hair-thin wires for coils, from linemen's safety buckles to the precious metal on relay contacts.

In joints and connections — soldered or welded, brazed or riveted — photomicrographs reveal flaws which would escape ordinary tests. They show if a batch of steel has the right structure to stand up in service; why a guy wire let go in a high wind or a filament snapped in a vacuum tube; how to make switchboard plugs last longer.

In their exploration of micro-structure, Bell Telephone Laboratories scientists have contributed importantly to the metallographic art. You enjoy the benefits of their work in the value and reliability of your telephone system, and the low cost of its service.

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